Package ‘gaussratiovegind’

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Title Distribution of Gaussian Ratios

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Description It is well known that the distribution of a Gaussian ratio
does not follow a Gaussian distribution.
The lack of awareness among users of vegetation indices about this
non-Gaussian nature could lead to incorrect statistical modeling
and interpretation.
This package provides tools to accurately handle and analyse such
ratios: density function, parameter estimation, simulation.
An example on the study of chlorophyll fluorescence can be found in
A. El Ghaziri et al. (2023) <doi:10.3390/rs15020528>.

License GPL (>= 3)

URL https://forgemia.inra.fr/imhorphen/gaussratiovegind

BugReports https://forgemia.inra.fr/imhorphen/gaussratiovegind/-/issues

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`dnormratio` *Ratio of two Gaussian distributions*

Description

Density of the ratio of two Gaussian distributions.

Usage

```
dnormratio(z, bet, rho, delta)
```

Arguments

`z`	length p numeric vector.
`bet, rho, delta`	numeric values. The parameters (β, ρ, δ) of the distribution, see Details.

Details

Let two independant random variables $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$.

If we denote $\beta = \frac{\mu_x}{\mu_y}$, $\rho = \frac{\sigma_y}{\sigma_x}$ and $\delta_y = \frac{\sigma_y}{\mu_y}$, the probability distribution function of the ratio $Z = \frac{X}{Y}$ is given by:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1+\rho^2 z^2)} \left[\exp\left(-\frac{\rho^2 \beta^2 + 1}{2\delta_y^2} \right) + \sqrt{\frac{\pi}{2}}\, q \,\mathrm{erf}\left(\frac{q}{\sqrt{2}} \right) \exp\left(-\frac{\rho^2 (z-\beta)^2}{2\delta_y^2 (1+\rho^2 z^2)} \right) \right]$$

$$\text{with } q = \frac{1 + \beta\rho^2 z}{\delta_y \sqrt{1+\rho^2 z^2}} \text{ and } \mathrm{erf}\left(\frac{q}{\sqrt{2}} \right) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{q}{\sqrt{2}}} \exp\left(-t^2\right) dt$$

Value

Numeric: the value of density.

Author(s)

Pierre Santagostini, Angélina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528

Díaz-Francés, E., Rubio, F.J., On the existence of a normal approximation to the distribution of the ratio of two independent normal random variables. Stat Papers 54, 309–323 (2013). doi:10.1007/s0036201204292

See Also

`rnormratio()`: sample simulation.

`estparnormratio()`: parameter estimation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
dnormratio(0, bet = beta1, rho = rho1, delta = delta1)
dnormratio(0.5, bet = beta1, rho = rho1, delta = delta1)
curve(dnormratio(x, bet = beta1, rho = rho1, delta = delta1), from = -0.1, to = 0.7)

# Second example
beta2 <- 2
rho2 <- 2
delta2 <- 2
dnormratio(0, bet = beta2, rho = rho2, delta = delta2)
dnormratio(0.5, bet = beta2, rho = rho2, delta = delta2)
curve(dnormratio(x, bet = beta2, rho = rho2, delta = delta2), from = -0.1, to = 0.7)
```

`estparnormratio` *Estimation of the Parameters of a Normal Ratio Distribution*

Description

Estimation of the parameters of a ratio $Z = \frac{X}{Y}$, X and Y being two independant random variables distributed according to Gaussian distributions, using the EM (estimation-maximization) algorithm.

Usage

```
estparnormratio(z, eps = 1e-6)
```

Arguments

`z`	numeric matrix or data frame.
`eps`	numeric. Precision for the estimation of the parameters.

Details

Let a random variable: $Z = \frac{X}{Y}$,

X and Y being normally distributed: $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$.

The density probability of Z is:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1 + \rho^2 z^2)} \exp\left(-\frac{\rho^2 \beta^2 + 1}{2\delta_y^2}\right) {}_1F_1\left(1, \frac{1}{2}; \frac{1}{2\delta_y^2} \frac{(1 + \beta\rho^2 z)^2}{1 + \rho^2 z^2}\right)$$

with: $\beta = \frac{\mu_x}{\mu_y}$, $\rho = \frac{\sigma_y}{\sigma_x}$, $\delta_y = \frac{\sigma_y}{\mu_y}$.

and ${}_1F_1(a, b; x)$ is the confluent hypergeometric function (Kummer's function):

$${}_1F_1(a, b; x) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n} \frac{x^n}{n!}$$

The parameters β, ρ, δ_y of the Z distribution are estimated with the EM algorithm, as presented in El Ghaziri et al. The computation uses the kummerM function.

This uses an iterative algorithm.

The precision for the estimation of the parameters is given by the eps parameter.

Value

A list of 3 elements beta, rho, delta: the estimated parameters of the Z distribution $\hat{\beta}$, $\hat{\rho}$, $\hat{\delta}_y$, with two attributes attr(, "epsilon") (precision of the result) and attr(, "k") (number of iterations).

Author(s)

Pierre Santagostini, Angélina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528

See Also

dnormratio(): probability density of a normal ratio.

rnormratio(): sample simulation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22

set.seed(1234)
```

```
z1 <- rnormratio(800, bet = beta1, rho = rho1, delta = delta1)

estparnormratio(z1)

# Second example
beta2 <- 0.24
rho2 <- 4.21
delta2 <- 0.25

set.seed(1234)
z2 <- rnormratio(800, bet = beta2, rho = rho2, delta = delta2)

estparnormratio(z2)
```

`kummerM` *Confluent D-Hypergeometric Function*

Description

Computes the Kummer's function, or confluent hypergeometric function.

Usage

```
kummerM(a, b, z, eps = 1e-06)
```

Arguments

`a`	numeric.
`b`	numeric
`z`	numeric vector.
`eps`	numeric. Precision for the sum (default 1e-06).

Details

The Kummer's confluent hypergeometric function is given by:

$$ {}_1F_1\left(a, b; z\right) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n} \frac{z^n}{n!} $$

where $(z)_p$ is the Pochhammer symbol (see `pochhammer`).

The eps argument gives the required precision for its computation. It is the `attr(, "epsilon")` attribute of the returned value.

Value

A numeric value: the value of the Kummer's function, with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Angélina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528

`lnpochhammer` *Logarithm of the Pochhammer Symbol*

Description

Computes the logarithm of the Pochhammer symbol.

Usage

```
lnpochhammer(x, n)
```

Arguments

`x`	numeric.
`n`	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

So, if $n > 0$:

$$log\left((x)_n\right) = log(x) + log(x+1) + ... + log(x+n-1)$$

If $n = 0$, $log\left((x)_n\right) = log(1) = 0$

Value

Numeric value. The logarithm of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

pochhammer, kummerM

Examples

```
lnpochhammer(2, 0)
lnpochhammer(2, 1)
lnpochhammer(2, 3)
```

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pochhammer — *Pochhammer Symbol*

Description

Computes the Pochhammer symbol.

Usage

```
pochhammer(x, n)
```

Arguments

x	numeric.
n	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

Value

Numeric value. The value of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

lnpochhammer, kummerM

Examples

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```

`rnormratio` *Ratio of two Gaussian distributions*

Description

Simulate data from a ratio of two Gaussian distributions.

Usage

```
rnormratio(n, bet, rho, delta)
```

Arguments

`n`	integer. Number of observations. If `length(n)` > 1, the length is taken to be the nmber required.
`bet, rho, delta`	numeric values. The parameters (β, ρ, δ) of the distribution, see Details.

Details

Let two random variables $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$

with probability densities f_X and f_Y.

The parameters of the distribution of the ratio $Z = \frac{X}{Y}$ are: $\beta = \dfrac{\mu_x}{\mu_y}$, $\rho = \dfrac{\sigma_y}{\sigma_x}$, $\delta_y = \dfrac{\sigma_y}{\mu_y}$.

μ_x, σ_x, μ_y and σ_y are computed from β, ρ and δ_y (by fixing arbitrarily $\mu_x = 1$) and two random samples $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ are simulated.

Then $\left(\dfrac{x_1}{y_1}, \ldots, \dfrac{x_n}{y_n}\right)$ is returned.

Value

A numeric vector: the produced sample.

Author(s)

Pierre Santagostini, Angélina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528

See Also

dnormratio(): probability density of a normal ratio.

estparnormratio(): parameter estimation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
rnormratio(20, bet = beta1, rho = rho1, delta = delta1)

# Second example
beta2 <- 0.24
rho2 <- 4.21
delta2 <- 0.25
rnormratio(20, bet = beta2, rho = rho2, delta = delta2)
```

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