Package 'cluster'

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Priority recommended

Title ``Finding Groups in Data'': Cluster Analysis Extended Rousseeuw et
   al.

Description Methods for Cluster analysis. Much extended the original from
   Peter Rousseeuw, Anja Struyf and Mia Hubert,
   based on Kaufman and Rousseeuw (1990) ``Finding Groups in Data''.

Maintainer Martin Maechler <maechler@stat.math.ethz.ch>

Depends R (>= 3.5.0)

Imports graphics, grDevices, stats, utils

Suggests MASS, Matrix

SuggestsNote MASS: two examples using cov.rob() and mvrnorm(); Matrix
   tools for testing

Enhances mvoutlier, fpc, ellipse, sfsmisc

EnhancesNote xref-ed in man/*.Rd

LazyLoad yes

LazyData yes

ByteCompile yes

BuildResaveData no

License GPL (>= 2)

URL https://svn.r-project.org/R-packages/trunk/cluster/

NeedsCompilation yes

Author Martin Maechler [aut, cre] (ORCID:
   <https://orcid.org/0000-0002-8685-9910>),
   Peter Rousseeuw [aut] (Fortran original, ORCID:
   <https://orcid.org/0000-0002-3807-5353>),

Anja Struyf [aut] (S original),
Mia Hubert [aut] (S original, ORCID:
 <https://orcid.org/0000-0001-6398-4850>),
Kurt Hornik [trl, ctb] (port to R; maintenance(1999-2000), ORCID:
 <https://orcid.org/0000-0003-4198-9911>),
Matthias Studer [ctb],
Pierre Roudier [ctb],
Juan Gonzalez [ctb],
Kamil Kozlowski [ctb],
Erich Schubert [ctb] (fastpam options for pam(), ORCID:
 <https://orcid.org/0000-0001-9143-4880>),
Keefe Murphy [ctb] (volume.ellipsoid({d >= 3}))

Contents

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agnes   *Agglomerative Nesting (Hierarchical Clustering)*

Description

Computes agglomerative hierarchical clustering of the dataset.

Usage

```
agnes(x, diss = inherits(x, "dist"), metric = "euclidean",
      stand = FALSE, method = "average", par.method,
      keep.diss = n < 100, keep.data = !diss, trace.lev = 0)
```

Arguments

x   data matrix or data frame, or dissimilarity matrix, depending on the value of the
                diss argument.

                In case of a matrix or data frame, each row corresponds to an observation, and
                each column corresponds to a variable. All variables must be numeric. Missing
                values (NAs) are allowed.

In case of a dissimilarity matrix, x is typically the output of [daisy](#) or [dist](#).
Also a vector with length n*(n-1)/2 is allowed (where n is the number of ob-
servations), and will be interpreted in the same way as the output of the above-
mentioned functions. Missing values (NAs) are not allowed.

diss   logical flag: if TRUE (default for dist or dissimilarity objects), then x is
                  assumed to be a dissimilarity matrix. If FALSE, then x is treated as a matrix of
                  observations by variables.

metric   character string specifying the metric to be used for calculating dissimilarities
                  between observations. The currently available options are "euclidean" and
                  "manhattan". Euclidean distances are root sum-of-squares of differences, and
                  manhattan distances are the sum of absolute differences. If x is already a dis-
                  similarity matrix, then this argument will be ignored.

stand   logical flag: if TRUE, then the measurements in x are standardized before cal-
                  culating the dissimilarities. Measurements are standardized for each variable
                  (column), by subtracting the variable's mean value and dividing by the vari-
                  able's mean absolute deviation. If x is already a dissimilarity matrix, then this
                  argument will be ignored.

method   character string defining the clustering method. The six methods implemented
                  are "average" ([unweighted pair-]group [arithMetic] average method, aka 'UP-
                  GMA'), "single" (single linkage), "complete" (complete linkage), "ward"
                  (Ward's method), "weighted" (weighted average linkage, aka 'WPGMA'), its
                  generalization "flexible" which uses (a constant version of) the Lance-Williams
                  formula and the par.method argument, and "gaverage" a generalized "average"
                  aka "flexible UPGMA" method also using the Lance-Williams formula and par.method.
                  The default is "average".

par.method   If method is "flexible" or "gaverage", a numeric vector of length 1, 3, or 4,
                  (with a default for "gaverage"), see in the details section.

keep.diss, keep.data

                  logicals indicating if the dissimilarities and/or input data x should be kept in the
                  result. Setting these to FALSE can give much smaller results and hence even save
                  memory allocation *time*.

trace.lev   integer specifying a trace level for printing diagnostics during the algorithm.
                  Default 0 does not print anything; higher values print increasingly more.

Details

agnes is fully described in chapter 5 of Kaufman and Rousseeuw (1990). Compared to other
agglomerative clustering methods such as hclust, agnes has the following features: (a) it yields the
agglomerative coefficient (see [agnes.object](#)) which measures the amount of clustering structure
found; and (b) apart from the usual tree it also provides the banner, a novel graphical display (see
[plot.agnes](#)).

The agnes-algorithm constructs a hierarchy of clusterings.

At first, each observation is a small cluster by itself. Clusters are merged until only one large cluster
remains which contains all the observations. At each stage the two *nearest* clusters are combined to
form one larger cluster.

For method="average", the distance between two clusters is the average of the dissimilarities be-
tween the points in one cluster and the points in the other cluster.

In `method="single"`, we use the smallest dissimilarity between a point in the first cluster and a point in the second cluster (nearest neighbor method).

When `method="complete"`, we use the largest dissimilarity between a point in the first cluster and a point in the second cluster (furthest neighbor method).

The `method = "flexible"` allows (and requires) more details: The Lance-Williams formula specifies how dissimilarities are computed when clusters are agglomerated (equation (32) in K&R(1990), p.237). If clusters C_1 and C_2 are agglomerated into a new cluster, the dissimilarity between their union and another cluster Q is given by

$$D(C_1 \cup C_2, Q) = \alpha_1 * D(C_1, Q) + \alpha_2 * D(C_2, Q) + \beta * D(C_1, C_2) + \gamma * |D(C_1, Q) - D(C_2, Q)|,$$

where the four coefficients $(\alpha_1, \alpha_2, \beta, \gamma)$ are specified by the vector `par.method`, either directly as vector of length 4, or (more conveniently) if `par.method` is of length 1, say $= \alpha$, `par.method` is extended to give the "Flexible Strategy" (K&R(1990), p.236 f) with Lance-Williams coefficients $(\alpha_1 = \alpha_2 = \alpha, \beta = 1 - 2\alpha, \gamma = 0)$.
Also, if `length(par.method) == 3`, $\gamma = 0$ is set.

Care and expertise is probably needed when using `method = "flexible"` particularly for the case when `par.method` is specified of longer length than one. Since **cluster** version 2.0, choices leading to invalid `merge` structures now signal an error (from the C code already). The *weighted average* (`method="weighted"`) is the same as `method="flexible"`, `par.method = 0.5`. Further, `method= "single"` is equivalent to `method="flexible"`, `par.method = c(.5,.5,0,-.5)`, and `method="complete"` is equivalent to `method="flexible"`, `par.method = c(.5,.5,0,+.5)`.

The `method = "gaverage"` is a generalization of `"average"`, aka "flexible UPGMA" method, and is (a generalization of the approach) detailed in Belbin et al. (1992). As `"flexible"`, it uses the Lance-Williams formula above for dissimilarity updating, but with α_1 and α_2 not constant, but *proportional* to the *sizes* n_1 and n_2 of the clusters C_1 and C_2 respectively, i.e,

$$\alpha_j = \alpha_j' \frac{n_1}{n_1 + n_2},$$

where α_1', α_2' are determined from `par.method`, either directly as $(\alpha_1, \alpha_2, \beta, \gamma)$ or $(\alpha_1, \alpha_2, \beta)$ with $\gamma = 0$, or (less flexibly, but more conveniently) as follows:

Belbin et al proposed "flexible beta", i.e. the user would only specify β (as `par.method`), sensibly in
$$-1 \leq \beta < 1,$$
and β determines α_1' and α_2' as
$$\alpha_j' = 1 - \beta,$$
and $\gamma = 0$.

This β may be specified by `par.method` (as length 1 vector), and if `par.method` is not specified, a default value of -0.1 is used, as Belbin et al recommend taking a β value around -0.1 as a general agglomerative hierarchical clustering strategy.

Note that `method = "gaverage"`, `par.method = 0` (or `par.method = c(1,1,0,0)`) is equivalent to the `agnes()` default method `"average"`.

Value

an object of class `"agnes"` (which extends `"twins"`) representing the clustering. See `agnes.object` for details, and methods applicable.

BACKGROUND

Cluster analysis divides a dataset into groups (clusters) of observations that are similar to each other.

Hierarchical methods like agnes, diana, and mona construct a hierarchy of clusterings, with the number of clusters ranging from one to the number of observations.

Partitioning methods like pam, clara, and fanny require that the number of clusters be given by the user.

Author(s)

Method "gaverage" has been contributed by Pierre Roudier, Landcare Research, New Zealand.

References

Kaufman, L. and Rousseeuw, P.J. (1990). (=: "K&R(1990)") *Finding Groups in Data: An Introduction to Cluster Analysis*. Wiley, New York.

Anja Struyf, Mia Hubert and Peter J. Rousseeuw (1996) Clustering in an Object-Oriented Environment. *Journal of Statistical Software* **1**. doi:10.18637/jss.v001.i04

Struyf, A., Hubert, M. and Rousseeuw, P.J. (1997). Integrating Robust Clustering Techniques in S-PLUS, *Computational Statistics and Data Analysis*, **26**, 17–37.

Lance, G.N., and W.T. Williams (1966). A General Theory of Classifactory Sorting Strategies, I. Hierarchical Systems. *Computer J.* **9**, 373–380.

Belbin, L., Faith, D.P. and Milligan, G.W. (1992). A Comparison of Two Approaches to Beta-Flexible Clustering. *Multivariate Behavioral Research*, **27**, 417–433.

See Also

agnes.object, daisy, diana, dist, hclust, plot.agnes, twins.object.

Examples

```
data(votes.repub)
agn1 <- agnes(votes.repub, metric = "manhattan", stand = TRUE)
agn1
plot(agn1)

op <- par(mfrow=c(2,2))
agn2 <- agnes(daisy(votes.repub), diss = TRUE, method = "complete")
plot(agn2)
## alpha = 0.625 ==> beta = -1/4  is "recommended" by some
agnS <- agnes(votes.repub, method = "flexible", par.method = 0.625)
plot(agnS)
par(op)

## "show" equivalence of three "flexible" special cases
d.vr <- daisy(votes.repub)
a.wgt  <- agnes(d.vr, method = "weighted")
a.sing <- agnes(d.vr, method = "single")
a.comp <- agnes(d.vr, method = "complete")
```

```
iC <- -(6:7) # not using 'call' and 'method' for comparisons
stopifnot(
  all.equal(a.wgt [iC], agnes(d.vr, method="flexible", par.method = 0.5)[iC])   ,
  all.equal(a.sing[iC], agnes(d.vr, method="flex", par.method= c(.5,.5,0, -.5))[iC]),
  all.equal(a.comp[iC], agnes(d.vr, method="flex", par.method= c(.5,.5,0, +.5))[iC]))

## Exploring the dendrogram structure
(d2 <- as.dendrogram(agn2)) # two main branches
d2[[1]] # the first branch
d2[[2]] # the 2nd one  { 8 + 42  = 50 }
d2[[1]][[1]]# first sub-branch of branch 1 .. and shorter form
identical(d2[[c(1,1)]],
          d2[[1]][[1]])
## a "textual picture" of the dendrogram :
str(d2)

data(agriculture)

## Plot similar to Figure 7 in ref
## Not run: plot(agnes(agriculture), ask = TRUE)


data(animals)
aa.a  <- agnes(animals) # default method = "average"
aa.ga <- agnes(animals, method = "gaverage")
op <- par(mfcol=1:2, mgp=c(1.5, 0.6, 0), mar=c(.1+ c(4,3,2,1)),
          cex.main=0.8)
plot(aa.a,  which.plots = 2)
plot(aa.ga, which.plots = 2)
par(op)


## Show how "gaverage" is a "generalized average":
aa.ga.0 <- agnes(animals, method = "gaverage", par.method = 0)
stopifnot(all.equal(aa.ga.0[iC], aa.a[iC]))
```

---

agnes.object   *Agglomerative Nesting (AGNES) Object*

Description

The objects of class "agnes" represent an agglomerative hierarchical clustering of a dataset.

Value

A legitimate agnes object is a list with the following components:

order   a vector giving a permutation of the original observations to allow for plotting,
               in the sense that the branches of a clustering tree will not cross.

order.lab   a vector similar to order, but containing observation labels instead of observation numbers. This component is only available if the original observations were labelled.

height   a vector with the distances between merging clusters at the successive stages.

ac   the agglomerative coefficient, measuring the clustering structure of the dataset.

                  For each observation i, denote by m(i) its dissimilarity to the first cluster it is merged with, divided by the dissimilarity of the merger in the final step of the algorithm. The ac is the average of all 1 - m(i). It can also be seen as the average width (or the percentage filled) of the banner plot. Because ac grows with the number of observations, this measure should not be used to compare datasets of very different sizes.

merge   an (n-1) by 2 matrix, where n is the number of observations. Row i of merge describes the merging of clusters at step i of the clustering. If a number j in the row is negative, then the single observation |j| is merged at this stage. If j is positive, then the merger is with the cluster formed at stage j of the algorithm.

diss   an object of class "dissimilarity" (see dissimilarity.object), representing the total dissimilarity matrix of the dataset.

data   a matrix containing the original or standardized measurements, depending on the stand option of the function agnes. If a dissimilarity matrix was given as input structure, then this component is not available.

GENERATION

This class of objects is returned from agnes.

METHODS

The "agnes" class has methods for the following generic functions: print, summary, plot, and as.dendrogram.

In addition, cutree(x, *) can be used to "cut" the dendrogram in order to produce cluster assignments.

INHERITANCE

The class "agnes" inherits from "twins". Therefore, the generic functions pltree and as.hclust are available for agnes objects. After applying as.hclust(), all *its* methods are available, of course.

See Also

agnes, diana, as.hclust, hclust, plot.agnes, twins.object.

cutree.

Examples

```
data(agriculture)
ag.ag <- agnes(agriculture)
class(ag.ag)
pltree(ag.ag) # the dendrogram

## cut the dendrogram -> get cluster assignments:
(ck3 <- cutree(ag.ag, k = 3))
(ch6 <- cutree(as.hclust(ag.ag), h = 6))
stopifnot(identical(unname(ch6), ck3))
```

---

agriculture   *European Union Agricultural Workforces*

Description

Gross National Product (GNP) per capita and percentage of the population working in agriculture for each country belonging to the European Union in 1993.

Usage

```
data(agriculture)
```

Format

A data frame with 12 observations on 2 variables:

[, 1]	x	numeric	per capita GNP
[, 2]	y	numeric	percentage in agriculture

The row names of the data frame indicate the countries.

Details

The data seem to show two clusters, the "more agricultural" one consisting of Greece, Portugal, Spain, and Ireland.

Source

Eurostat (European Statistical Agency, 1994): *Cijfers en feiten: Een statistisch portret van de Europese Unie*.

References

see those in [agnes](#).

See Also

[agnes](#), [daisy](#), [diana](#).

Examples

```
data(agriculture)

## Compute the dissimilarities using Euclidean metric and without
## standardization
daisy(agriculture, metric = "euclidean", stand = FALSE)

## 2nd plot is similar to Figure 3 in Struyf et al (1996)
plot(pam(agriculture, 2))

## Plot similar to Figure 7 in Struyf et al (1996)
## Not run: plot(agnes(agriculture), ask = TRUE)


## Plot similar to Figure 8 in Struyf et al (1996)
## Not run: plot(diana(agriculture), ask = TRUE)
```

---

animals   *Attributes of Animals*

Description

This data set considers 6 binary attributes for 20 animals.

Usage

```
data(animals)
```

Format

A data frame with 20 observations on 6 variables:

[, 1]	war	warm-blooded
[, 2]	fly	can fly
[, 3]	ver	vertebrate
[, 4]	end	endangered
[, 5]	gro	live in groups
[, 6]	hai	have hair

All variables are encoded as 1 = 'no', 2 = 'yes'.

Details

This dataset is useful for illustrating monothetic (only a single variable is used for each split) hierarchical clustering.

Source

Leonard Kaufman and Peter J. Rousseeuw (1990): *Finding Groups in Data* (pp 297ff). New York: Wiley.

References

see Struyf, Hubert & Rousseeuw (1996), in `agnes`.

Examples

```
data(animals)
apply(animals,2, table) # simple overview

ma <- mona(animals)
ma
## Plot similar to Figure 10 in Struyf et al (1996)
plot(ma)
```

---

bannerplot   *Plot Banner (of Hierarchical Clustering)*

Description

Draws a "banner", i.e. basically a horizontal `barplot` visualizing the (agglomerative or divisive) hierarchical clustering or an other binary dendrogram structure.

Usage

```
bannerplot(x, w = rev(x$height), fromLeft = TRUE,
           main=NULL, sub=NULL, xlab = "Height",  adj = 0,
           col = c(2, 0), border = 0, axes = TRUE, frame.plot = axes,
           rev.xax = !fromLeft, xax.pretty = TRUE,
           labels = NULL, nmax.lab = 35, max.strlen = 5,
           yax.do = axes && length(x$order) <= nmax.lab,
           yaxRight = fromLeft, y.mar = 2.4 + max.strlen/2.5, ...)
```

Arguments

x	a list with components order, order.lab and height when w, the next argument is not specified.
w	non-negative numeric vector of bar widths.
fromLeft	logical, indicating if the banner is from the left or not.
main, sub	main and sub titles, see `title`.
xlab	x axis label (with 'correct' default e.g. for plot.agnes).
adj	passed to `title`(main,sub) for string adjustment.
col	vector of length 2, for two horizontal segments.

border	color for bar border; now defaults to background (no border).
axes	logical indicating if axes (and labels) should be drawn at all.
frame.plot	logical indicating the banner should be framed; mainly used when border = 0 (as per default).
rev.xax	logical indicating if the x axis should be reversed (as in plot.diana).
xax.pretty	logical or integer indicating if [pretty]()() should be used for the x axis. xax.pretty = FALSE is mainly for back compatibility.
labels	labels to use on y-axis; the default is constructed from x.
nmax.lab	integer indicating the number of labels which is considered too large for single-name labelling the banner plot.
max.strlen	positive integer giving the length to which strings are truncated in banner plot labeling.
yax.do	logical indicating if a y axis and banner labels should be drawn.
yaxRight	logical indicating if the y axis is on the right or left.
y.mar	positive number specifying the margin width to use when banners are labeled (along a y-axis). The default adapts to the string width and optimally would also dependend on the font.
...	graphical parameters (see [par]()) may also be supplied as arguments to this function.

Note

This is mainly a utility called from [plot.agnes](), [plot.diana]() and [plot.mona]().

Author(s)

Martin Maechler (from original code of Kaufman and Rousseeuw).

Examples

```
data(agriculture)
bannerplot(agnes(agriculture), main = "Bannerplot")
```

---

chorSub   *Subset of C-horizon of Kola Data*

Description

This is a small rounded subset of the C-horizon data [chorizon]() from package **mvoutlier**.

Usage

```
data(chorSub)
```

Format

A data frame with 61 observations on 10 variables. The variables contain scaled concentrations of chemical elements.

Details

This data set was produced from `chorizon` via these statements:

```
data(chorizon, package = "mvoutlier")
chorSub <- round(100*scale(chorizon[,101:110]))[190:250,]
storage.mode(chorSub) <- "integer"
colnames(chorSub) <- gsub("_.*", '', colnames(chorSub))
```

Source

Kola Project (1993-1998)

See Also

[chorizon](#) in package **mvoutlier** and other Kola data in the same package.

Examples

```
data(chorSub)
summary(chorSub)
pairs(chorSub, gap= .1)# some outliers
```

---

clara   *Clustering Large Applications*

Description

Computes a `"clara"` object, a [list](#) representing a clustering of the data into k clusters.

Usage

```
clara(x, k, metric = c("euclidean", "manhattan", "jaccard"),
      stand = FALSE, cluster.only = FALSE, samples = 5,
      sampsize = min(n, 40 + 2 * k), trace = 0, medoids.x = TRUE,
      keep.data = medoids.x, rngR = FALSE, pamLike = FALSE, correct.d = TRUE)
```

Arguments

x	data matrix or data frame, each row corresponds to an observation, and each column corresponds to a variable. All variables must be numeric (or logical). Missing values (NAs) are allowed.
k	integer, the number of clusters. It is required that $0 < k < n$ where n is the number of observations (i.e., n = nrow(x)).
metric	character string specifying the metric to be used for calculating dissimilarities between observations. The currently available options are "euclidean", "manhattan", "jaccard".
	Euclidean distances are root sum-of-squares of differences, and manhattan distances are the sum of absolute differences.
stand	logical, indicating if the measurements in x are standardized before calculating the dissimilarities. Measurements are standardized for each variable (column), by subtracting the variable's mean value and dividing by the variable's mean absolute deviation.
cluster.only	logical; if true, only the clustering will be computed and returned, see details.
samples	integer, say N, the number of samples to be drawn from the dataset. The default, N = 5, is rather small for historical (and now back compatibility) reasons and we *recommend to set* samples *an order of magnitude larger*.
sampsize	integer, say j, the number of observations in each sample. sampsize should be higher than the number of clusters (k) and at most the number of observations (n = nrow(x)). While computational effort is proportional to j^2, see note below, it may still be advisable to set j =sampsize to a *larger* value than the (historical) default.
trace	integer indicating a *trace level* for diagnostic output during the algorithm.
medoids.x	logical indicating if the medoids should be returned, identically to some rows of the input data x. If FALSE, keep.data must be false as well, and the medoid indices, i.e., row numbers of the medoids will still be returned (i.med component), and the algorithm saves space by needing one copy less of x.
keep.data	logical indicating if the (*scaled* if stand is true) data should be kept in the result. Setting this to FALSE saves memory (and hence time), but disables clusplot()ing of the result. Use medoids.x = FALSE to save even more memory.
rngR	logical indicating if R's random number generator should be used instead of the primitive clara()-builtin one. If true, this also means that each call to clara() returns a different result – though only slightly different in good situations.
pamLike	logical indicating if the "swap" phase (see [pam](pam), in C code) should use the same algorithm as [pam](pam)(). Note that from Kaufman and Rousseeuw's description this *should* have been true always, but as the original Fortran code and the subsequent port to C has always contained a small one-letter change (a typo according to Martin Maechler) with respect to PAM, the default, pamLike = FALSE has been chosen to remain back compatible rather than "PAM compatible".
correct.d	logical or integer indicating that—only in the case of NAs present in x—the correct distance computation should be used instead of the wrong formula which has been present in the original Fortran code and been in use up to early 2016. Because the new correct formula is not back compatible, for the time being, a warning is signalled in this case, unless the user explicitly specifies correct.d.

Details

clara (for "euclidean" and "manhattan") is fully described in chapter 3 of Kaufman and Rousseeuw (1990). Compared to other partitioning methods such as pam, it can deal with much larger datasets. Internally, this is achieved by considering sub-datasets of fixed size (sampsize) such that the time and storage requirements become linear in n rather than quadratic.

Each sub-dataset is partitioned into k clusters using the same algorithm as in [pam](). Once k representative objects have been selected from the sub-dataset, each observation of the entire dataset is assigned to the nearest medoid.

The mean (equivalent to the sum) of the dissimilarities of the observations to their closest medoid is used as a measure of the quality of the clustering. The sub-dataset for which the mean (or sum) is minimal, is retained. A further analysis is carried out on the final partition.

Each sub-dataset is forced to contain the medoids obtained from the best sub-dataset until then. Randomly drawn observations are added to this set until sampsize has been reached.

When cluster.only is true, the result is simply a (possibly named) integer vector specifying the clustering, i.e.,
clara(x,k, cluster.only=TRUE) is the same as
clara(x,k)$clustering but computed more efficiently.

Value

If cluster.only is false (as by default), an object of class "clara" representing the clustering. See [clara.object]() for details.

If cluster.only is true, the result is the "clustering", an integer vector of length n with entries from 1:k.

Note

By default, the random sampling is implemented with a *very* simple scheme (with period $2^{16} = 65536$) inside the Fortran code, independently of R's random number generation, and as a matter of fact, deterministically. Alternatively, we recommend setting rngR = TRUE which uses R's random number generators. Then, clara() results are made reproducible typically by using [set.seed]() before calling clara.

The storage requirement of clara computation (for small k) is about $O(n \times p) + O(j^2)$ where $j = $ sampsize, and $(n, p) = $ dim(x). The CPU computing time (again assuming small k) is about $O(n \times p \times j^2 \times N)$, where $N = $ samples.

For "small" datasets, the function [pam]() can be used directly. What can be considered *small*, is really a function of available computing power, both memory (RAM) and speed. Originally (1990), "small" meant less than 100 observations; in 1997, the authors said *"small (say with fewer than 200 observations)"*; as of 2006, you can use [pam]() with several thousand observations.

Author(s)

Kaufman and Rousseeuw (see [agnes]()), originally. Metric "jaccard": Kamil Kozlowski (@ownedoutcomes.com) and Kamil Jadeszko. All arguments from trace on, and most R documentation and all tests by Martin Maechler.

See Also

agnes for background and references; clara.object, pam, partition.object, plot.partition.

Examples

```
## generate 500 objects, divided into 2 clusters.
x <- rbind(cbind(rnorm(200,0,8), rnorm(200,0,8)),
           cbind(rnorm(300,50,8), rnorm(300,50,8)))
clarax <- clara(x, 2, samples=50)
clarax
clarax$clusinfo
## using pamLike=TRUE  gives the same (apart from the 'call'):
all.equal(clarax[-8],
          clara(x, 2, samples=50, pamLike = TRUE)[-8])
plot(clarax)

## cluster.only = TRUE -- save some memory/time :
clclus <- clara(x, 2, samples=50, cluster.only = TRUE)
stopifnot(identical(clclus, clarax$clustering))


## 'xclara' is an artificial data set with 3 clusters of 1000 bivariate
## objects each.
data(xclara)
(clx3 <- clara(xclara, 3))
## "better" number of samples
cl.3 <- clara(xclara, 3, samples=100)
## but that did not change the result here:
stopifnot(cl.3$clustering == clx3$clustering)
## Plot similar to Figure 5 in Struyf et al (1996)
## Not run: plot(clx3, ask = TRUE)


## Try 100 times *different* random samples -- for reliability:
nSim <- 100
nCl <- 3 # = no.classes
set.seed(421)# (reproducibility)
cl <- matrix(NA,nrow(xclara), nSim)
for(i in 1:nSim)
   cl[,i] <- clara(xclara, nCl, medoids.x = FALSE, rngR = TRUE)$clustering
tcl <- apply(cl,1, tabulate, nbins = nCl)
## those that are not always in same cluster (5 out of 3000 for this seed):
(iDoubt <- which(apply(tcl,2, function(n) all(n < nSim))))
if(length(iDoubt)) { # (not for all seeds)
  tabD <- tcl[,iDoubt, drop=FALSE]
  dimnames(tabD) <- list(cluster = paste(1:nCl), obs = format(iDoubt))
  t(tabD) # how many times in which clusters
}
```

---

clara.object   *Clustering Large Applications (CLARA) Object*

Description

The objects of class "clara" represent a partitioning of a large dataset into clusters and are typically returned from [clara](#).

Value

A legitimate clara object is a list with the following components:

sample   labels or case numbers of the observations in the best sample, that is, the sample used by the clara algorithm for the final partition.

medoids   the medoids or representative objects of the clusters. It is a matrix with in each row the coordinates of one medoid. Possibly NULL, namely when the object resulted from clara(*, medoids.x=FALSE). Use the following i.med in that case.

i.med   the *indices* of the medoids above: medoids <- x[i.med,] where x is the original data matrix in clara(x,*).

clustering   the clustering vector, see [partition.object](#).

objective   the objective function for the final clustering of the entire dataset.

clusinfo   matrix, each row gives numerical information for one cluster. These are the cardinality of the cluster (number of observations), the maximal and average dissimilarity between the observations in the cluster and the cluster's medoid. The last column is the maximal dissimilarity between the observations in the cluster and the cluster's medoid, divided by the minimal dissimilarity between the cluster's medoid and the medoid of any other cluster. If this ratio is small, the cluster is well-separated from the other clusters.

diss   dissimilarity (maybe NULL), see [partition.object](#).

silinfo   list with silhouette width information for the best sample, see [partition.object](#).

call   generating call, see [partition.object](#).

data   matrix, possibibly standardized, or NULL, see [partition.object](#).

Methods, Inheritance

The "clara" class has methods for the following generic functions: print, summary.

The class "clara" inherits from "partition". Therefore, the generic functions plot and clusplot can be used on a clara object.

See Also

[clara](#), [dissimilarity.object](#), [partition.object](#), [plot.partition](#).

---

clusGap   *Gap Statistic for Estimating the Number of Clusters*

Description

clusGap() calculates a goodness of clustering measure, the "gap" statistic. For each number of clusters k, it compares $\log(W(k))$ with $E^*[\log(W(k))]$ where the latter is defined via bootstrapping, i.e., simulating from a reference (H_0) distribution, a uniform distribution on the hypercube determined by the ranges of x, after first centering, and then [svd](aka 'PCA')-rotating them when (as by default) spaceH0 = "scaledPCA".

maxSE(f, SE.f) determines the location of the **maximum** of f, taking a "1-SE rule" into account for the *SE* methods. The default method "firstSEmax" looks for the smallest k such that its value $f(k)$ is not more than 1 standard error away from the first local maximum. This is similar but not the same as "Tibs2001SEmax", Tibshirani et al's recommendation of determining the number of clusters from the gap statistics and their standard deviations.

Usage

```
clusGap(x, FUNcluster, K.max, B = 100, d.power = 1,
        spaceH0 = c("scaledPCA", "original"),
        verbose = interactive(), ...)

maxSE(f, SE.f,
      method = c("firstSEmax", "Tibs2001SEmax", "globalSEmax",
                 "firstmax", "globalmax"),
      SE.factor = 1)

## S3 method for class 'clusGap'
print(x, method = "firstSEmax", SE.factor = 1, ...)

## S3 method for class 'clusGap'
plot(x, type = "b", xlab = "k", ylab = expression(Gap[k]),
     main = NULL, do.arrows = TRUE,
     arrowArgs = list(col="red3", length=1/16, angle=90, code=3), ...)
```

Arguments

x	numeric matrix or [data.frame]().
FUNcluster	a [function]() which accepts as first argument a (data) matrix like x, second argument, say $k, k \geq 2$, the number of clusters desired, and returns a [list]() with a component named (or shortened to) cluster which is a vector of length n = nrow(x) of integers in 1:k determining the clustering or grouping of the n observations.
K.max	the maximum number of clusters to consider, must be at least two.
B	integer, number of Monte Carlo ("bootstrap") samples.

d.power	a positive integer specifying the power p which is applied to the euclidean distances ([dist](#)) before they are summed up to give $W(k)$. The default, d.power = 1, corresponds to the "historical" R implementation, whereas d.power = 2 corresponds to what Tibshirani et al had proposed. This was found by Juan Gonzalez, in 2016-02.
spaceH0	a [character](#) string specifying the space of the H_0 distribution (of *no* cluster). Both "scaledPCA" and "original" use a uniform distribution in a hyper cube and had been mentioned in the reference; "original" been added after a proposal (including code) by Juan Gonzalez.
verbose	integer or logical, determining if "progress" output should be printed. The default prints one bit per bootstrap sample.
...	(for clusGap():) optionally further arguments for FUNcluster(), see kmeans example below.
f	numeric vector of 'function values', of length K, whose ("1 SE respected") maximum we want.
SE.f	numeric vector of length K of standard errors of f.
method	character string indicating how the "optimal" number of clusters, $\hat{k}$, is computed from the gap statistics (and their standard deviations), or more generally how the location $\hat{k}$ of the maximum of f_k should be determined.

>   "globalmax": simply corresponds to the global maximum, i.e., is which.max(f)
>
>   "firstmax": gives the location of the first *local* maximum.
>
>   "Tibs2001SEmax": uses the criterion, Tibshirani et al (2001) proposed: "the smallest k such that $f(k) \geq f(k+1) - s_{k+1}$". Note that this chooses $k = 1$ when all standard deviations are larger than the differences $f(k+1) - f(k)$.
>
>   "firstSEmax": location of the first $f()$ value which is not smaller than the first *local* maximum minus SE.factor * SE.f[], i.e, within an "f S.E." range of that maximum (see also SE.factor).
>   This, the default, has been proposed by Martin Maechler in 2012, when adding clusGap() to the **cluster** package, after having seen the "globalSEmax" proposal (in code) and read the "Tibs2001SEmax" proposal.
>
>   "globalSEmax": (used in Dudoit and Fridlyand (2002), supposedly following Tibshirani's proposition): location of the first $f()$ value which is not smaller than the *global* maximum minus SE.factor * SE.f[], i.e, within an "f S.E." range of that maximum (see also SE.factor).
>
> See the examples for a comparison in a simple case.

SE.factor	[When method contains "SE"] Determining the optimal number of clusters, Tibshirani et al. proposed the "1 S.E."-rule. Using an SE.factor f, the "f S.E."-rule is used, more generally.
type, xlab, ylab, main	
	arguments with the same meaning as in [plot.default](#)(), with different default.
do.arrows	logical indicating if (1 SE -)"error bars" should be drawn, via [arrows](#)().
arrowArgs	a list of arguments passed to [arrows](#)(); the default, notably angle and code, provide a style matching usual error bars.

Details

The main result `<res>$Tab[,"gap"]` of course is from bootstrapping aka Monte Carlo simulation and hence random, or equivalently, depending on the initial random seed (see `set.seed()`). On the other hand, in our experience, using `B = 500` gives quite precise results such that the gap plot is basically unchanged after an another run.

Value

`clusGap(..)` returns an object of S3 class `"clusGap"`, basically a list with components

Tab	a matrix with `K.max` rows and 4 columns, named "logW", "E.logW", "gap", and "SE.sim", where gap = `E.logW` − `logW`, and `SE.sim` corresponds to the standard error of gap, `SE.sim[k]`=s_k, where $s_k := \sqrt{1 + 1/B} sd^*(gap_j)$, and $sd^*()$ is the standard deviation of the simulated ("bootstrapped") gap values.
call	the `clusGap(..)` `call`.
spaceH0	the `spaceH0` argument (`match.arg`()ed).
n	number of observations, i.e., `nrow(x)`.
B	input B
FUNcluster	input function `FUNcluster`

Author(s)

This function is originally based on the functions gap of former (Bioconductor) package **SAGx** by Per Broberg, `gapStat()` from former package **SLmisc** by Matthias Kohl and ideas from gap() and its methods of package **lga** by Justin Harrington.

The current implementation is by Martin Maechler.

The implementation of `spaceH0 = "original"` is based on code proposed by Juan Gonzalez.

References

Tibshirani, R., Walther, G. and Hastie, T. (2001). Estimating the number of data clusters via the Gap statistic. *Journal of the Royal Statistical Society B*, **63**, 411–423.

Tibshirani, R., Walther, G. and Hastie, T. (2000). Estimating the number of clusters in a dataset via the Gap statistic. Technical Report. Stanford.

Dudoit, S. and Fridlyand, J. (2002) A prediction-based resampling method for estimating the number of clusters in a dataset. *Genome Biology* **3**(7). doi:10.1186/gb200237research0036

Per Broberg (2006). SAGx: Statistical Analysis of the GeneChip. R package version 1.9.7. https://bioconductor.org/packages/3.12/bioc/html/SAGx.html Deprecated and removed from Bioc ca. 2022

See Also

`silhouette` for a much simpler less sophisticated goodness of clustering measure.

`cluster.stats()` in package **fpc** for alternative measures.

Examples

```
### --- maxSE() methods ------------------------------------------
(mets <- eval(formals(maxSE)$method))
fk <- c(2,3,5,4,7,8,5,4)
sk <- c(1,1,2,1,1,3,1,1)/2
## use plot.clusGap():
plot(structure(class="clusGap", list(Tab = cbind(gap=fk, SE.sim=sk))))
## Note that 'firstmax' and 'globalmax' are always at 3 and 6 :
sapply(c(1/4, 1,2,4), function(SEf)
        sapply(mets, function(M) maxSE(fk, sk, method = M, SE.factor = SEf)))

### --- clusGap() ------------------------------------------
## ridiculously nicely separated clusters in 3 D :
x <- rbind(matrix(rnorm(150,           sd = 0.1), ncol = 3),
           matrix(rnorm(150, mean = 1, sd = 0.1), ncol = 3),
           matrix(rnorm(150, mean = 2, sd = 0.1), ncol = 3),
           matrix(rnorm(150, mean = 3, sd = 0.1), ncol = 3))

## Slightly faster way to use pam (see below)
pam1 <- function(x,k) list(cluster = pam(x,k, cluster.only=TRUE))

## We do not recommend using hier.clustering here, but if you want,
## there is  factoextra::hcut () or a cheap version of it
hclusCut <- function(x, k, d.meth = "euclidean", ...)
   list(cluster = cutree(hclust(dist(x, method=d.meth), ...), k=k))

## You can manually set it before running this :    doExtras <- TRUE  # or  FALSE
if(!(exists("doExtras") && is.logical(doExtras)))
  doExtras <- cluster:::doExtras()

if(doExtras) {
  ## Note we use  B = 60 in the following examples to keep them "speedy".
  ## ---- rather keep the default B = 500 for your analysis!

  ## note we can  pass 'nstart = 20' to kmeans() :
  gskmn <- clusGap(x, FUNcluster = kmeans, nstart = 20, K.max = 8, B = 60)
  gskmn #-> its print() method
  plot(gskmn, main = "clusGap(., FUNcluster = kmeans, n.start=20, B= 60)")
  set.seed(12); system.time(
    gsPam0 <- clusGap(x, FUNcluster = pam, K.max = 8, B = 60)
  )
  set.seed(12); system.time(
    gsPam1 <- clusGap(x, FUNcluster = pam1, K.max = 8, B = 60)
  )
  ## and show that it gives the "same":
  not.eq <- c("call", "FUNcluster"); n <- names(gsPam0)
  eq <- n[!(n %in% not.eq)]
  stopifnot(identical(gsPam1[eq], gsPam0[eq]))
  print(gsPam1, method="globalSEmax")
  print(gsPam1, method="globalmax")

  print(gsHc <- clusGap(x, FUNcluster = hclusCut, K.max = 8, B = 60))
```

```
}# end {doExtras}

gs.pam.RU <- clusGap(ruspini, FUNcluster = pam1, K.max = 8, B = 60)
gs.pam.RU
plot(gs.pam.RU, main = "Gap statistic for the 'ruspini' data")
mtext("k = 4 is best .. and  k = 5  pretty close")

## This takes a minute..
## No clustering ==> k = 1 ("one cluster") should be optimal:
Z <- matrix(rnorm(256*3), 256,3)
gsP.Z <- clusGap(Z, FUNcluster = pam1, K.max = 8, B = 200)
plot(gsP.Z, main = "clusGap(<iid_rnorm_p=3>)  ==> k = 1  cluster is optimal")
gsP.Z
```

---

clusplot   *Bivariate Cluster Plot (of a Partitioning Object)*

Description

Draws a 2-dimensional "clusplot" (clustering plot) on the current graphics device. The generic
function has a default and a partition method.

Usage

```
clusplot(x, ...)

## S3 method for class 'partition'
clusplot(x, main = NULL, dist = NULL, ...)
```

Arguments

x	an R object, here, specifically an object of class "partition", e.g. created by one of the functions [pam](pam), clara, or fanny.
main	title for the plot; when NULL (by default), a title is constructed, using x$call.
dist	when x does not have a diss nor a data component, e.g., for [pam](pam)(dist(*), keep.diss=FALSE), dist must specify the dissimilarity for the clusplot.
...	optional arguments passed to methods, notably the clusplot.default method (except for the diss one) may also be supplied to this function. Many graphical parameters (see [par](par)) may also be supplied as arguments here.

Details

The clusplot.partition() method relies on clusplot.default.

If the clustering algorithms pam, fanny and clara are applied to a data matrix of observations-by-
variables then a clusplot of the resulting clustering can always be drawn. When the data matrix

contains missing values and the clustering is performed with [pam](#) or [fanny](#), the dissimilarity matrix will be given as input to clusplot. When the clustering algorithm [clara](#) was applied to a data matrix with [NA](#)s then clusplot() will replace the missing values as described in [clusplot.default](#), because a dissimilarity matrix is not available.

Value

For the partition (and default) method: An invisible list with components Distances and Shading, as for [clusplot.default](#), see there.

Side Effects

a 2-dimensional clusplot is created on the current graphics device.

See Also

[clusplot.default](#) for references; [partition.object](#), [pam](#), [pam.object](#), [clara](#), [clara.object](#), [fanny](#), [fanny.object](#), [par](#).

Examples

```
 ## For more, see ?clusplot.default

## generate 25 objects, divided into 2 clusters.
x <- rbind(cbind(rnorm(10,0,0.5), rnorm(10,0,0.5)),
           cbind(rnorm(15,5,0.5), rnorm(15,5,0.5)))
clusplot(pam(x, 2))
## add noise, and try again :
x4 <- cbind(x, rnorm(25), rnorm(25))
clusplot(pam(x4, 2))
```

---

clusplot.default   *Bivariate Cluster Plot (clusplot) Default Method*

Description

Creates a bivariate plot visualizing a partition (clustering) of the data. All observation are represented by points in the plot, using principal components or multidimensional scaling. Around each cluster an ellipse is drawn.

Usage

```
## Default S3 method:
clusplot(x, clus, diss = FALSE,
         s.x.2d = mkCheckX(x, diss), stand = FALSE,
         lines = 2, shade = FALSE, color = FALSE,
         labels= 0, plotchar = TRUE,
         col.p = "dark green", col.txt = col.p,
         col.clus = if(color) c(2, 4, 6, 3) else 5, cex = 1, cex.txt = cex,
```

```
            span = TRUE,
            add = FALSE,
            xlim = NULL, ylim = NULL,
            main = paste("CLUSPLOT(", deparse1(substitute(x)),")"),
            sub = paste("These two components explain",
                round(100 * var.dec, digits = 2), "% of the point variability."),
            xlab = "Component 1", ylab = "Component 2",
            verbose = getOption("verbose"),
            ...)
```

Arguments

x
: matrix or data frame, or dissimilarity matrix, depending on the value of the diss argument.

  In case of a matrix (alike), each row corresponds to an observation, and each column corresponds to a variable. All variables must be numeric. Missing values (NAs) are allowed. They are replaced by the median of the corresponding variable. When some variables or some observations contain only missing values, the function stops with a warning message.

  In case of a dissimilarity matrix, x is the output of daisy or dist or a symmetric matrix. Also, a vector of length $n * (n-1)/2$ is allowed (where n is the number of observations), and will be interpreted in the same way as the output of the above-mentioned functions. Missing values (NAs) are not allowed.

clus
: a vector of length n representing a clustering of x. For each observation the vector lists the number or name of the cluster to which it has been assigned. clus is often the clustering component of the output of [pam](pam), fanny or clara.

diss
: logical indicating if x will be considered as a dissimilarity matrix or a matrix of observations by variables (see x arugment above).

s.x.2d
: a list with components named x (a $n \times 2$ matrix; typically something like principal components of original data), labs and var.dec.

stand
: logical flag: if true, then the representations of the n observations in the 2-dimensional plot are standardized.

lines
: integer out of 0, 1, 2, used to obtain an idea of the distances between ellipses. The distance between two ellipses E1 and E2 is measured along the line connecting the centers $m1$ and $m2$ of the two ellipses.

  In case E1 and E2 overlap on the line through $m1$ and $m2$, no line is drawn. Otherwise, the result depends on the value of lines: If

  lines = 0, no distance lines will appear on the plot;

  lines = 1, the line segment between $m1$ and $m2$ is drawn;

  lines = 2, a line segment between the boundaries of E1 and E2 is drawn (along the line connecting $m1$ and $m2$).

shade
: logical flag: if TRUE, then the ellipses are shaded in relation to their density. The density is the number of points in the cluster divided by the area of the ellipse.

color
: logical flag: if TRUE, then the ellipses are colored with respect to their density. With increasing density, the colors are light blue, light green, red and purple. To

	see these colors on the graphics device, an appropriate color scheme should be selected (we recommend a white background).
labels	integer code, currently one of 0,1,2,3,4 and 5. If

> **labels= 0,** no labels are placed in the plot;
>
> **labels= 1,** points and ellipses can be identified in the plot (see [identify]);
>
> **labels= 2,** all points and ellipses are labelled in the plot;
>
> **labels= 3,** only the points are labelled in the plot;
>
> **labels= 4,** only the ellipses are labelled in the plot.
>
> **labels= 5,** the ellipses are labelled in the plot, and points can be identified.

> The levels of the vector clus are taken as labels for the clusters. The labels of the points are the rownames of x if x is matrix like. Otherwise (diss = TRUE), x is a vector, point labels can be attached to x as a "Labels" attribute (attr(x,"Labels")), as is done for the output of [daisy].
>
> A possible [names] attribute of clus will not be taken into account.

plotchar	logical flag: if TRUE, then the plotting symbols differ for points belonging to different clusters.
span	logical flag: if TRUE, then each cluster is represented by the ellipse with smallest area containing all its points. (This is a special case of the minimum volume ellipsoid.) If FALSE, the ellipse is based on the mean and covariance matrix of the same points. While this is faster to compute, it often yields a much larger ellipse. There are also some special cases: When a cluster consists of only one point, a tiny circle is drawn around it. When the points of a cluster fall on a straight line, span=FALSE draws a narrow ellipse around it and span=TRUE gives the exact line segment.
add	logical indicating if ellipses (and labels if labels is true) should be *added* to an already existing plot. If false, neither a [title] or sub title, see sub, is written.
col.p	color code(s) used for the observation points.
col.txt	color code(s) used for the labels (if labels >= 2).
col.clus	color code for the ellipses (and their labels); only one if color is false (as per default).
cex, cex.txt	character **ex**pansion (size), for the point symbols and point labels, respectively.
xlim, ylim	numeric vectors of length 2, giving the x- and y- ranges as in [plot.default].
main	main title for the plot; by default, one is constructed.
sub	sub title for the plot; by default, one is constructed.
xlab, ylab	x- and y- axis labels for the plot, with defaults.
verbose	a logical indicating, if there should be extra diagnostic output; mainly for 'debugging'.
...	Further graphical parameters may also be supplied, see [par].

Details

clusplot uses function calls [princomp](*, cor = (ncol(x) > 2)) or [cmdscale](*, add=TRUE), re-
spectively, depending on diss being false or true. These functions are data reduction techniques to
represent the data in a bivariate plot.

Ellipses are then drawn to indicate the clusters. The further layout of the plot is determined by the
optional arguments.

Value

An invisible list with components:

Distances   When lines is 1 or 2 we optain a k by k matrix (k is the number of clusters).
                 The element in [i,j] is the distance between ellipse i and ellipse j.
                 If lines = 0, then the value of this component is NA.

Shading   A vector of length k (where k is the number of clusters), containing the amount
                 of shading per cluster. Let y be a vector where element i is the ratio between the
                 number of points in cluster i and the area of ellipse i. When the cluster i is a line
                 segment, y[i] and the density of the cluster are set to NA. Let z be the sum of all
                 the elements of y without the NAs. Then we put shading = y/z *37 + 3 .

Side Effects

a visual display of the clustering is plotted on the current graphics device.

Note

When we have 4 or fewer clusters, then the color=TRUE gives every cluster a different color. When
there are more than 4 clusters, clusplot uses the function [pam](pam) to cluster the densities into 4 groups
such that ellipses with nearly the same density get the same color. col.clus specifies the colors
used.

The col.p and col.txt arguments, added for R, are recycled to have length the number of obser-
vations. If col.p has more than one value, using color = TRUE can be confusing because of a mix
of point and ellipse colors.

References

Pison, G., Struyf, A. and Rousseeuw, P.J. (1999) Displaying a Clustering with CLUSPLOT, *Com-
putational Statistics and Data Analysis*, **30**, 381–392.

Kaufman, L. and Rousseeuw, P.J. (1990). *Finding Groups in Data: An Introduction to Cluster
Analysis.* Wiley, New York.

Struyf, A., Hubert, M. and Rousseeuw, P.J. (1997). Integrating Robust Clustering Techniques in
S-PLUS, *Computational Statistics and Data Analysis*, **26**, 17-37.

See Also

princomp, cmdscale, [pam](pam), clara, daisy, [par](par), identify, [cov.mve](cov.mve), clusplot.partition.

Examples

```
## plotting votes.diss(dissimilarity) in a bivariate plot and
## partitioning into 2 clusters
data(votes.repub)
votes.diss <- daisy(votes.repub)
pamv <- pam(votes.diss, 2, diss = TRUE)
clusplot(pamv, shade = TRUE)
## is the same as
votes.clus <- pamv$clustering
clusplot(votes.diss, votes.clus, diss = TRUE, shade = TRUE)
## Now look at components 3 and 2 instead of 1 and 2:
str(cMDS <- cmdscale(votes.diss, k=3, add=TRUE))
clusplot(pamv, s.x.2d = list(x=cMDS$points[, c(3,2)],
                             labs=rownames(votes.repub), var.dec=NA),
         shade = TRUE, col.p = votes.clus,
         sub="", xlab = "Component 3", ylab = "Component 2")

clusplot(pamv, col.p = votes.clus, labels = 4)# color points and label ellipses
# "simple" cheap ellipses: larger than minimum volume:
# here they are *added* to the previous plot:
clusplot(pamv, span = FALSE, add = TRUE, col.clus = "midnightblue")

## Setting a small *label* size:
clusplot(votes.diss, votes.clus, diss = TRUE, labels = 3, cex.txt = 0.6)

if(dev.interactive()) { #  uses identify() *interactively* :
  clusplot(votes.diss, votes.clus, diss = TRUE, shade = TRUE, labels = 1)
  clusplot(votes.diss, votes.clus, diss = TRUE, labels = 5)# ident. only points
}

## plotting iris (data frame) in a 2-dimensional plot and partitioning
## into 3 clusters.
data(iris)
iris.x <- iris[, 1:4]
cl3 <- pam(iris.x, 3)$clustering
op <- par(mfrow= c(2,2))
clusplot(iris.x, cl3, color = TRUE)
U <- par("usr")
## zoom in :
rect(0,-1, 2,1, border = "orange", lwd=2)
clusplot(iris.x, cl3, color = TRUE, xlim = c(0,2), ylim = c(-1,1))
box(col="orange",lwd=2); mtext("sub region", font = 4, cex = 2)
##  or zoom out :
clusplot(iris.x, cl3, color = TRUE, xlim = c(-4,4), ylim = c(-4,4))
mtext("'super' region", font = 4, cex = 2)
rect(U[1],U[3], U[2],U[4], lwd=2, lty = 3)

# reset graphics
par(op)
```

| coef.hclust | *Agglomerative / Divisive Coefficient for 'hclust' Objects* |

Description

Computes the "agglomerative coefficient" (aka "divisive coefficient" for [diana](#)), measuring the clustering structure of the dataset.

For each observation i, denote by $m(i)$ its dissimilarity to the first cluster it is merged with, divided by the dissimilarity of the merger in the final step of the algorithm. The agglomerative coefficient is the average of all $1 - m(i)$. It can also be seen as the average width (or the percentage filled) of the banner plot.

coefHier() directly interfaces to the underlying C code, and "proves" that *only* object$heights is needed to compute the coefficient.

Because it grows with the number of observations, this measure should not be used to compare datasets of very different sizes.

Usage

```
coefHier(object)
coef.hclust(object, ...)
## S3 method for class 'hclust'
coef(object, ...)
## S3 method for class 'twins'
coef(object, ...)
```

Arguments

object	an object of class "hclust" or "twins", i.e., typically the result of [hclust](#)(.),[agnes](#)(.), or [diana](#)(.).
	Since coef.hclust only uses object$heights, and object$merge, object can be any list-like object with appropriate merge and heights components.
	For coefHier, even only object$heights is needed.
...	currently unused potential further arguments

Value

a number specifying the *agglomerative* (or *divisive* for diana objects) coefficient as defined by Kaufman and Rousseeuw, see [agnes.object](#) $ ac or [diana.object](#) $ dc.

Examples

```
data(agriculture)
aa <- agnes(agriculture)
coef(aa) # really just extracts aa$ac
coef(as.hclust(aa))# recomputes
coefHier(aa)        # ditto
```

---

daisy   *Dissimilarity Matrix Calculation*

Description

Compute all the pairwise dissimilarities (distances) between observations in the data set. The original variables may be of mixed types. In that case, or whenever metric = "gower" is set, a generalization of Gower's formula is used, see 'Details' below.

Usage

```
daisy(x, metric = c("euclidean", "manhattan", "gower"),
      stand = FALSE, type = list(), weights = rep.int(1, p),
      warnBin = warnType, warnAsym = warnType, warnConst = warnType,
      warnType = TRUE)
```

Arguments

x   numeric matrix or data frame, of dimension $n \times p$, say. Dissimilarities will be computed between the rows of x. Columns of mode numeric (i.e. all columns when x is a matrix) will be recognized as interval scaled variables, columns of class factor will be recognized as nominal variables, and columns of class ordered will be recognized as ordinal variables. Other variable types should be specified with the type argument. Missing values (NAs) are allowed.

metric   character string specifying the metric to be used. The currently available options are "euclidean" (the default), "manhattan" and "gower".
Euclidean distances are root sum-of-squares of differences, and manhattan distances are the sum of absolute differences.

                 "Gower's distance" is chosen by metric "gower" or automatically if some columns of x are not numeric. Also known as Gower's coefficient (1971), expressed as a dissimilarity, this implies that a particular standardisation will be applied to each variable, and the "distance" between two units is the sum of all the variable-specific distances, see the details section.

stand   logical flag: if TRUE, then the measurements in x are standardized before calculating the dissimilarities. Measurements are standardized for each variable (column), by subtracting the variable's mean value and dividing by the variable's mean absolute deviation.

                 If not all columns of x are numeric, stand will be ignored and Gower's standardization (based on the range) will be applied in any case, see argument metric, above, and the details section.

type   list for specifying some (or all) of the types of the variables (columns) in x. The list may contain the following components:

                 "asymm" **A**symmetric binary variable, aka "A" in result Types, see dissimilarity.object.
                 "symm" **S**ymmetric binary variable, aka "S".

"factor" **N**ominal – the default for [factor](#) variables, aka "N". When the factor has 2 levels, this is equivalent to type = "S" for a (symmetric) binary variable.

"ordered" **O**rdinal – the default for [ordered](#) (factor) variables, aka "O", see [dissimilarity.object](#).

"logratio" ratio scaled numeric variables that are to be logarithmically transformed ([log10](#)) and then treated as numeric ("I"): must be *positive* numeric variable.

"ordratio" "ra**T**io"-like variable to be treated as [ordered](#) (using the factor codes unclass(as.ordered(x[,j]))), aka "T".

"numeric"/"integer" **I**nterval scaled – the **default** for all numeric (incl integer) columns of x, aka "I" in result Types, see [dissimilarity.object](#).

Each component is a (character or numeric) vector, containing either the names or the numbers of the corresponding columns of x.

Variables not mentioned in type are interpreted as usual, see argument x, and also 'default' above. Consequently, the default type = list() may often be sufficient.

weights   an optional numeric vector of length p(=ncol(x)); to be used in "case 2" (mixed variables, or metric = "gower"), specifying a weight for each variable (x[,k]) instead of 1 in Gower's original formula.

warnBin, warnAsym, warnConst

logicals indicating if the corresponding type checking warnings should be signalled (when found).

warnType   logical indicating if *all* the type checking warnings should be active or not.

Details

The original version of daisy is fully described in chapter 1 of Kaufman and Rousseeuw (1990). Compared to [dist](#) whose input must be numeric variables, the main feature of daisy is its ability to handle other variable types as well (e.g. nominal, ordinal, (a)symmetric binary) even when different types occur in the same data set.

The handling of nominal, ordinal, and (a)symmetric binary data is achieved by using the general dissimilarity coefficient of Gower (1971). If x contains any columns of these data-types, both arguments metric and stand will be ignored and Gower's coefficient will be used as the metric. This can also be activated for purely numeric data by metric = "gower". With that, each variable (column) is first standardized by dividing each entry by the range of the corresponding variable, after subtracting the minimum value; consequently the rescaled variable has range $[0, 1]$, exactly.

Note that setting the type to symm (symmetric binary) gives the same dissimilarities as using *nominal* (which is chosen for non-ordered factors) only when no missing values are present, and more efficiently.

Note that daisy signals a warning when 2-valued numerical variables do not have an explicit type specified, because the reference authors recommend to consider using "asymm"; the warning may be silenced by warnBin = FALSE.

In the daisy algorithm, missing values in a row of x are not included in the dissimilarities involving that row. There are two main cases,

1. If all variables are interval scaled (and `metric` is *not* "gower"), the metric is "euclidean", and n_g is the number of columns in which neither row i and j have NAs, then the dissimilarity d(i,j) returned is $\sqrt{p/n_g}$ (p =ncol(x)) times the Euclidean distance between the two vectors of length n_g shortened to exclude NAs. The rule is similar for the "manhattan" metric, except that the coefficient is p/n_g. If $n_g = 0$, the dissimilarity is NA.

2. When some variables have a type other than interval scaled, or if `metric = "gower"` is specified, the dissimilarity between two rows is the weighted mean of the contributions of each variable. Specifically,

$$d_{ij} = d(i,j) = \frac{\sum_{k=1}^{p} w_k \delta_{ij}^{(k)} d_{ij}^{(k)}}{\sum_{k=1}^{p} w_k \delta_{ij}^{(k)}}.$$

In other words, d_{ij} is a weighted mean of $d_{ij}^{(k)}$ with weights $w_k \delta_{ij}^{(k)}$, where w_k= `weigths[k]`, $\delta_{ij}^{(k)}$ is 0 or 1, and $d_{ij}^{(k)}$, the k-th variable contribution to the total distance, is a distance between `x[i,k]` and `x[j,k]`, see below.

The 0-1 weight $\delta_{ij}^{(k)}$ becomes zero when the variable `x[,k]` is missing in either or both rows (i and j), or when the variable is asymmetric binary and both values are zero. In all other situations it is 1.

The contribution $d_{ij}^{(k)}$ of a nominal or binary variable to the total dissimilarity is 0 if both values are equal, 1 otherwise. The contribution of other variables is the absolute difference of both values, divided by the total range of that variable. Note that "standard scoring" is applied to ordinal variables, i.e., they are replaced by their integer codes `1:K`. Note that this is not the same as using their ranks (since there typically are ties).

As the individual contributions $d_{ij}^{(k)}$ are in $[0,1]$, the dissimilarity d_{ij} will remain in this range. If all weights $w_k \delta_{ij}^{(k)}$ are zero, the dissimilarity is set to NA.

Value

an object of class "dissimilarity" containing the dissimilarities among the rows of x. This is typically the input for the functions pam, fanny, agnes or diana. For more details, see dissimilarity.object.

Background

Dissimilarities are used as inputs to cluster analysis and multidimensional scaling. The choice of metric may have a large impact.

Author(s)

Anja Struyf, Mia Hubert, and Peter and Rousseeuw, for the original version.
Martin Maechler improved the NA handling and type specification checking, and extended functionality to metric = "gower" and the optional weights argument.

References

Gower, J. C. (1971) A general coefficient of similarity and some of its properties, *Biometrics* **27**, 857–874.

Kaufman, L. and Rousseeuw, P.J. (1990) *Finding Groups in Data: An Introduction to Cluster Analysis*. Wiley, New York.

Struyf, A., Hubert, M. and Rousseeuw, P.J. (1997) Integrating Robust Clustering Techniques in S-PLUS, *Computational Statistics and Data Analysis* **26**, 17–37.

See Also

[dissimilarity.object](), [dist](), [pam](), [fanny](), [clara](), [agnes](), [diana]().

Examples

```
data(agriculture)
## Example 1 in ref:
##  Dissimilarities using Euclidean metric and without standardization
d.agr <- daisy(agriculture, metric = "euclidean", stand = FALSE)
d.agr
as.matrix(d.agr)[,"DK"] # via as.matrix.dist(.)
## compare with
as.matrix(daisy(agriculture, metric = "gower"))

## Example 2 in reference, extended  ---  different ways of "mixed" / "gower":

example(flower) # -> data(flower) *and* provide 'flowerN'

summary(d0    <- daisy(flower))  # -> the first 3 {0,1} treated as *N*ominal
summary(dS123 <- daisy(flower,  type = list(symm = 1:3))) # first 3 treated as *S*ymmetric
stopifnot(dS123 == d0) # i.e.,  *S*ymmetric <==> *N*ominal {for 2-level factor}
summary(dNS123<- daisy(flowerN, type = list(symm = 1:3)))
stopifnot(dS123 == d0)
## by default, however ...
summary(dA123 <- daisy(flowerN)) # .. all 3 logicals treated *A*symmetric binary (w/ warning)
summary(dA3   <- daisy(flower, type = list(asymm = 3)))
summary(dA13  <- daisy(flower, type = list(asymm = c(1, 3), ordratio = 7)))
## Mixing variable *names* and column numbers (failed in the past):
summary(dfl3 <- daisy(flower, type = list(asymm = c("V1", "V3"), symm= 2,
                                          ordratio= 7, logratio= 8)))

## If we'd treat the first 3 as simple {0,1}
Nflow <- flower
Nflow[,1:3] <- lapply(flower[,1:3], function(f) as.integer(as.character(f)))
summary(dN <- daisy(Nflow)) # w/ warning: treated binary .. 1:3 as interval
## Still, using Euclidean/Manhattan distance for {0-1} *is* identical to treating them as "N" :
stopifnot(dN == d0)
stopifnot(dN == daisy(Nflow, type = list(symm = 1:3))) # or as "S"
```

---

diana   *DIvisive ANAlysis Clustering*

Description

Computes a divisive hierarchical clustering of the dataset returning an object of class diana.

Usage

```
diana(x, diss = inherits(x, "dist"), metric = "euclidean", stand = FALSE,
      stop.at.k = FALSE,
      keep.diss = n < 100, keep.data = !diss, trace.lev = 0)
```

Arguments

x
: data matrix or data frame, or dissimilarity matrix or object, depending on the value of the diss argument.

  In case of a matrix or data frame, each row corresponds to an observation, and each column corresponds to a variable. All variables must be numeric. Missing values (NAs) *are* allowed.

  In case of a dissimilarity matrix, x is typically the output of daisy or dist. Also a vector of length n*(n-1)/2 is allowed (where n is the number of observations), and will be interpreted in the same way as the output of the above-mentioned functions. Missing values (NAs) are *not* allowed.

diss
: logical flag: if TRUE (default for dist or dissimilarity objects), then x will be considered as a dissimilarity matrix. If FALSE, then x will be considered as a matrix of observations by variables.

metric
: character string specifying the metric to be used for calculating dissimilarities between observations.

  The currently available options are "euclidean" and "manhattan". Euclidean distances are root sum-of-squares of differences, and manhattan distances are the sum of absolute differences. If x is already a dissimilarity matrix, then this argument will be ignored.

stand
: logical; if true, the measurements in x are standardized before calculating the dissimilarities. Measurements are standardized for each variable (column), by subtracting the variable's mean value and dividing by the variable's mean absolute deviation. If x is already a dissimilarity matrix, then this argument will be ignored.

stop.at.k
: logical or integer, FALSE by default. Otherwise must be integer, say k, in $\{1, 2, .., n\}$, specifying that the diana algorithm should stop early. Non-default NOT YET IMPLEMENTED.

keep.diss, keep.data
: logicals indicating if the dissimilarities and/or input data x should be kept in the result. Setting these to FALSE can give much smaller results and hence even save memory allocation *time*.

trace.lev
: integer specifying a trace level for printing diagnostics during the algorithm. Default 0 does not print anything; higher values print increasingly more.

Details

diana is fully described in chapter 6 of Kaufman and Rousseeuw (1990). It is probably unique in computing a divisive hierarchy, whereas most other software for hierarchical clustering is agglomerative. Moreover, diana provides (a) the divisive coefficient (see diana.object) which measures the amount of clustering structure found; and (b) the banner, a novel graphical display (see plot.diana).

The diana-algorithm constructs a hierarchy of clusterings, starting with one large cluster containing all n observations. Clusters are divided until each cluster contains only a single observation.

At each stage, the cluster with the largest diameter is selected. (The diameter of a cluster is the largest dissimilarity between any two of its observations.)

To divide the selected cluster, the algorithm first looks for its most disparate observation (i.e., which has the largest average dissimilarity to the other observations of the selected cluster). This observation initiates the "splinter group". In subsequent steps, the algorithm reassigns observations that are closer to the "splinter group" than to the "old party". The result is a division of the selected cluster into two new clusters.

Value

an object of class "diana" representing the clustering; this class has methods for the following generic functions: print, summary, plot.

Further, the class "diana" inherits from "twins". Therefore, the generic function pltree can be used on a diana object, and as.hclust and as.dendrogram methods are available.

A legitimate diana object is a list with the following components:

order	a vector giving a permutation of the original observations to allow for plotting, in the sense that the branches of a clustering tree will not cross.
order.lab	a vector similar to order, but containing observation labels instead of observation numbers. This component is only available if the original observations were labelled.
height	a vector with the diameters of the clusters prior to splitting.
dc	the divisive coefficient, measuring the clustering structure of the dataset. For each observation i, denote by $d(i)$ the diameter of the last cluster to which it belongs (before being split off as a single observation), divided by the diameter of the whole dataset. The dc is the average of all $1 - d(i)$. It can also be seen as the average width (or the percentage filled) of the banner plot. Because dc grows with the number of observations, this measure should not be used to compare datasets of very different sizes.
merge	an (n-1) by 2 matrix, where n is the number of observations. Row i of merge describes the split at step n-i of the clustering. If a number j in row r is negative, then the single observation $|j|$ is split off at stage n-r. If j is positive, then the cluster that will be splitted at stage n-j (described by row j), is split off at stage n-r.
diss	an object of class "dissimilarity", representing the total dissimilarity matrix of the dataset.
data	a matrix containing the original or standardized measurements, depending on the stand option of the function agnes. If a dissimilarity matrix was given as input structure, then this component is not available.

See Also

agnes also for background and references; cutree (and as.hclust) for grouping extraction; daisy, dist, plot.diana, twins.object.

Examples

```
data(votes.repub)
dv <- diana(votes.repub, metric = "manhattan", stand = TRUE)
print(dv)
plot(dv) #-> plot.diana()  {w/ its own help + examples}

## Cut into 2 groups:
dv2 <- cutree(as.hclust(dv), k = 2)
table(dv2) # 8 and 42 group members
rownames(votes.repub)[dv2 == 1]

## For two groups, does the metric matter ?
dv0 <- diana(votes.repub, stand = TRUE) # default: Euclidean
dv.2 <- cutree(as.hclust(dv0), k = 2)
table(dv2 == dv.2)## identical group assignments

str(as.dendrogram(dv0)) # {via as.dendrogram.twins() method}

data(agriculture)
## Plot similar to Figure 8 in ref
## Not run: plot(diana(agriculture), ask = TRUE)
```

---

dissimilarity.object   *Dissimilarity Matrix Object*

Description

Objects of class "dissimilarity" representing the dissimilarity matrix of a dataset.

Value

The dissimilarity matrix is symmetric, and hence its lower triangle (column wise) is represented as a vector to save storage space. If the object, is called do, and n the number of observations, i.e., n <- attr(do, "Size"), then for $i < j <= n$, the dissimilarity between (row) i and j is do[n*(i-1) - i*(i-1)/2 + j-i]. The length of the vector is $n * (n - 1)/2$, i.e., of order n^2.

"dissimilarity" objects also inherit from class [dist](#) and can use dist methods, in particular, [as.matrix](#), such that d_{ij} from above is just as.matrix(do)[i,j].

The object has the following attributes:

Size	the number of observations in the dataset.
Metric	the metric used for calculating the dissimilarities. Possible values are "euclidean", "manhattan", "mixed" (if variables of different types were present in the dataset), and "unspecified".
Labels	optionally, contains the labels, if any, of the observations of the dataset.
NA.message	optionally, if a dissimilarity could not be computed, because of too many missing values for some observations of the dataset.

Types   when a mixed metric was used, the types for each variable as one-letter codes,
                 see also type in [daisy]():

      A: Asymmetric binary

      S: Symmetric binary

      N: Nominal (factor)

      O: Ordinal (ordered factor)

      I: Interval scaled, possibly after log transform "logratio" (numeric)

      T: ra**T**io treated as ordered

GENERATION

daisy returns this class of objects. Also the functions pam, clara, fanny, agnes, and diana return
a dissimilarity object, as one component of their return objects.

METHODS

The "dissimilarity" class has methods for the following generic functions: print, summary.

See Also

daisy, dist, [pam](pam), clara, fanny, agnes, diana.

---

ellipsoidhull   *Compute the Ellipsoid Hull or Spanning Ellipsoid of a Point Set*

Description

Compute the "ellipsoid hull" or "spanning ellipsoid", i.e. the ellipsoid of minimal volume ('area'
in 2D) such that all given points lie just inside or on the boundary of the ellipsoid.

Usage

```
ellipsoidhull(x, tol=0.01, maxit=5000,
              ret.wt = FALSE, ret.sqdist = FALSE, ret.pr = FALSE)
## S3 method for class 'ellipsoid'
print(x, digits = max(1, getOption("digits") - 2), ...)
```

Arguments

x   the n p-dimensional points asnumeric $n \times p$ matrix.

tol   convergence tolerance for Titterington's algorithm. Setting this to much smaller
                 values may drastically increase the number of iterations needed, and you may
                 want to increas maxit as well.

maxit   integer giving the maximal number of iteration steps for the algorithm.

`ret.wt, ret.sqdist, ret.pr`

> logicals indicating if additional information should be returned, `ret.wt` specifying the *weights*, `ret.sqdist` the *squared distances* and `ret.pr` the final **pr**obabilities in the algorithms.

`digits, ...`   the usual arguments to `print` methods.

Details

The "spanning ellipsoid" algorithm is said to stem from Titterington(1976), in Pison et al (1999) who use it for `clusplot.default`.

The problem can be seen as a special case of the "Min.Vol." ellipsoid of which a more more flexible and general implementation is `cov.mve` in the MASS package.

Value

an object of class `"ellipsoid"`, basically a `list` with several components, comprising at least

`cov` $p \times p$ *covariance* matrix description the ellipsoid.

`loc` p-dimensional location of the ellipsoid center.

`d2`   average squared radius. Further, $d2 = t^2$, where t is "the value of a t-statistic on the ellipse boundary" (from `ellipse` in the **ellipse** package), and hence, more usefully, d2 = qchisq(alpha, df = p), where `alpha` is the confidence level for p-variate normally distributed data with location and covariance `loc` and `cov` to lie inside the ellipsoid.

`wt`   the vector of weights iff `ret.wt` was true.

`sqdist`   the vector of squared distances iff `ret.sqdist` was true.

`prob`   the vector of algorithm probabilities iff `ret.pr` was true.

`it`   number of iterations used.

`tol, maxit`   just the input argument, see above.

`eps`   the achieved tolerance which is the maximal squared radius minus p.

`ierr`   error code as from the algorithm; `0` means *ok*.

`conv`   logical indicating if the converged. This is defined as `it < maxit && ierr == 0`.

Author(s)

Martin Maechler did the present class implementation; Rousseeuw et al did the underlying original code.

References

Pison, G., Struyf, A. and Rousseeuw, P.J. (1999) Displaying a Clustering with CLUSPLOT, *Computational Statistics and Data Analysis*, **30**, 381–392.

D.M. Titterington (1976) Algorithms for computing D-optimal design on finite design spaces. In *Proc.\ of the 1976 Conf.\ on Information Science and Systems*, 213–216; John Hopkins University.

See Also

predict.ellipsoid which is also the predict method for ellipsoid objects. volume.ellipsoid
for an example of 'manual' ellipsoid object construction;
further ellipse from package **ellipse** and ellipsePoints from package **sfsmisc**.

chull for the convex hull, clusplot which makes use of this; cov.mve.

Examples

```
x <- rnorm(100)
xy <- unname(cbind(x, rnorm(100) + 2*x + 10))
exy. <- ellipsoidhull(xy)
exy. # >> calling print.ellipsoid()

plot(xy, main = "ellipsoidhull(<Gauss data>) -- 'spanning points'")
lines(predict(exy.), col="blue")
points(rbind(exy.$loc), col = "red", cex = 3, pch = 13)

exy <- ellipsoidhull(xy, tol = 1e-7, ret.wt = TRUE, ret.sqdist = TRUE)
str(exy) # had small 'tol', hence many iterations
(ii <- which(zapsmall(exy $ wt) > 1e-6))
## --> only about 4 to 6  "spanning ellipsoid" points
round(exy$wt[ii],3); sum(exy$wt[ii]) # weights summing to 1
points(xy[ii,], pch = 21, cex = 2,
       col="blue", bg = adjustcolor("blue",0.25))
```

---

fanny   *Fuzzy Analysis Clustering*

Description

Computes a fuzzy clustering of the data into k clusters.

Usage

```
fanny(x, k, diss = inherits(x, "dist"), memb.exp = 2,
      metric = c("euclidean", "manhattan", "SqEuclidean"),
      stand = FALSE, iniMem.p = NULL, cluster.only = FALSE,
      keep.diss = !diss && !cluster.only && n < 100,
      keep.data = !diss && !cluster.only,
      maxit = 500, tol = 1e-15, trace.lev = 0)
```

Arguments

x   data matrix or data frame, or dissimilarity matrix, depending on the value of the
             diss argument.

             In case of a matrix or data frame, each row corresponds to an observation, and
             each column corresponds to a variable. All variables must be numeric. Missing
             values (NAs) are allowed.

In case of a dissimilarity matrix, x is typically the output of daisy or dist. Also a vector of length n*(n-1)/2 is allowed (where n is the number of observations), and will be interpreted in the same way as the output of the above-mentioned functions. Missing values (NAs) are not allowed.

k	integer giving the desired number of clusters. It is required that $0 < k < n/2$ where n is the number of observations.
diss	logical flag: if TRUE (default for dist or dissimilarity objects), then x is assumed to be a dissimilarity matrix. If FALSE, then x is treated as a matrix of observations by variables.
memb.exp	number r strictly larger than 1 specifying the *membership exponent* used in the fit criterion; see the 'Details' below. Default: 2 which used to be hardwired inside FANNY.
metric	character string specifying the metric to be used for calculating dissimilarities between observations. Options are "euclidean" (default), "manhattan", and "SqEuclidean". Euclidean distances are root sum-of-squares of differences, and manhattan distances are the sum of absolute differences, and "SqEuclidean", the *squared* euclidean distances are sum-of-squares of differences. Using this last option is equivalent (but somewhat slower) to computing so called "fuzzy C-means". If x is already a dissimilarity matrix, then this argument will be ignored.
stand	logical; if true, the measurements in x are standardized before calculating the dissimilarities. Measurements are standardized for each variable (column), by subtracting the variable's mean value and dividing by the variable's mean absolute deviation. If x is already a dissimilarity matrix, then this argument will be ignored.
iniMem.p	numeric $n \times k$ matrix or NULL (by default); can be used to specify a starting membership matrix, i.e., a matrix of non-negative numbers, each row summing to one.
cluster.only	logical; if true, no silhouette information will be computed and returned, see details.
keep.diss, keep.data	logicals indicating if the dissimilarities and/or input data x should be kept in the result. Setting these to FALSE can give smaller results and hence also save memory allocation *time*.
maxit, tol	maximal number of iterations and default tolerance for convergence (relative convergence of the fit criterion) for the FANNY algorithm. The defaults maxit = 500 and tol = 1e-15 used to be hardwired inside the algorithm.
trace.lev	integer specifying a trace level for printing diagnostics during the C-internal algorithm. Default 0 does not print anything; higher values print increasingly more.

Details

In a fuzzy clustering, each observation is "spread out" over the various clusters. Denote by u_{iv} the membership of observation i to cluster v.

The memberships are nonnegative, and for a fixed observation i they sum to 1. The particular method fanny stems from chapter 4 of Kaufman and Rousseeuw (1990) (see the references in daisy) and has been extended by Martin Maechler to allow user specified memb.exp, iniMem.p, maxit, tol, etc.

Fanny aims to minimize the objective function

$$\sum_{v=1}^{k} \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} u_{iv}^{r} u_{jv}^{r} d(i,j)}{2 \sum_{j=1}^{n} u_{jv}^{r}}$$

where n is the number of observations, k is the number of clusters, r is the membership exponent memb.exp and $d(i,j)$ is the dissimilarity between observations i and j.

Note that $r \to 1$ gives increasingly crisper clusterings whereas $r \to \infty$ leads to complete fuzzyness. K&R(1990), p.191 note that values too close to 1 can lead to slow convergence. Further note that even the default, $r = 2$ can lead to complete fuzzyness, i.e., memberships $u_{iv} \equiv 1/k$. In that case a warning is signalled and the user is advised to chose a smaller memb.exp $(= r)$.

Compared to other fuzzy clustering methods, fanny has the following features: (a) it also accepts a dissimilarity matrix; (b) it is more robust to the spherical cluster assumption; (c) it provides a novel graphical display, the silhouette plot (see plot.partition).

Value

an object of class "fanny" representing the clustering. See fanny.object for details.

See Also

agnes for background and references; fanny.object, partition.object, plot.partition, daisy, dist.

Examples

```
## generate 10+15 objects in two clusters, plus 3 objects lying
## between those clusters.
x <- rbind(cbind(rnorm(10, 0, 0.5), rnorm(10, 0, 0.5)),
           cbind(rnorm(15, 5, 0.5), rnorm(15, 5, 0.5)),
           cbind(rnorm( 3,3.2,0.5), rnorm( 3,3.2,0.5)))
fannyx <- fanny(x, 2)
## Note that observations 26:28 are "fuzzy" (closer to # 2):
fannyx
summary(fannyx)
plot(fannyx)

(fan.x.15 <- fanny(x, 2, memb.exp = 1.5)) # 'crispier' for obs. 26:28
(fanny(x, 2, memb.exp = 3))               # more fuzzy in general

data(ruspini)
f4 <- fanny(ruspini, 4)
stopifnot(rle(f4$clustering)$lengths == c(20,23,17,15))
plot(f4, which = 1)
## Plot similar to Figure 6 in Stryuf et al (1996)
plot(fanny(ruspini, 5))
```

fanny.object	*Fuzzy Analysis (FANNY) Object*

Description

The objects of class `"fanny"` represent a fuzzy clustering of a dataset.

Value

A legitimate `fanny` object is a list with the following components:

membership	matrix containing the memberships for each pair consisting of an observation and a cluster.
memb.exp	the membership exponent used in the fitting criterion.
coeff	Dunn's partition coefficient $F(k)$ of the clustering, where k is the number of clusters. $F(k)$ is the sum of all *squared* membership coefficients, divided by the number of observations. Its value is between $1/k$ and 1.
	The normalized form of the coefficient is also given. It is defined as $(F(k) - 1/k)/(1 - 1/k)$, and ranges between 0 and 1. A low value of Dunn's coefficient indicates a very fuzzy clustering, whereas a value close to 1 indicates a near-crisp clustering.
clustering	the clustering vector of the nearest crisp clustering, see [partition.object](#).
k.crisp	integer $(\leq k)$ giving the number of *crisp* clusters; can be less than k, where it's recommended to decrease memb.exp.
objective	named vector containing the minimal value of the objective function reached by the FANNY algorithm and the relative convergence tolerance tol used.
convergence	named vector with iterations, the number of iterations needed and converged indicating if the algorithm converged (in maxit iterations within convergence tolerance tol).
diss	an object of class `"dissimilarity"`, see [partition.object](#).
call	generating call, see [partition.object](#).
silinfo	list with silhouette information of the nearest crisp clustering, see [partition.object](#).
data	matrix, possibibly standardized, or NULL, see [partition.object](#).

GENERATION

These objects are returned from [fanny](#).

METHODS

The `"fanny"` class has methods for the following generic functions: `print`, `summary`.

INHERITANCE

The class `"fanny"` inherits from `"partition"`. Therefore, the generic functions `plot` and `clusplot` can be used on a fanny object.

See Also

`fanny`, `print.fanny`, `dissimilarity.object`, `partition.object`, `plot.partition`.

---

flower   *Flower Characteristics*

Description

8 characteristics for 18 popular flowers.

Usage

```
data(flower)
```

Format

A data frame with 18 observations on 8 variables:

[, "V1"]	factor	winters
[, "V2"]	factor	shadow
[, "V3"]	factor	tubers
[, "V4"]	factor	color
[, "V5"]	ordered	soil
[, "V6"]	ordered	preference
[, "V7"]	numeric	height
[, "V8"]	numeric	distance

V1 winters, is binary and indicates whether the plant may be left in the garden when it freezes.

V2 shadow, is binary and shows whether the plant needs to stand in the shadow.

V3 tubers, is asymmetric binary and distinguishes between plants with tubers and plants that grow in any other way.

V4 color, is nominal and specifies the flower's color (1 = white, 2 = yellow, 3 = pink, 4 = red, 5 = blue).

V5 soil, is ordinal and indicates whether the plant grows in dry (1), normal (2), or wet (3) soil.

V6 preference, is ordinal and gives someone's preference ranking going from 1 to 18.

V7 height, is interval scaled, the plant's height in centimeters.

V8 distance, is interval scaled, the distance in centimeters that should be left between the plants.

References

Struyf, Hubert and Rousseeuw (1996), see `agnes`.

Examples

```
data(flower)
str(flower) # factors, ordered, numeric

## "Nicer" version (less numeric more self explainable) of 'flower':
flowerN <- flower
colnames(flowerN) <- c("winters", "shadow", "tubers", "color",
                       "soil", "preference", "height", "distance")
for(j in 1:3) flowerN[,j] <- (flowerN[,j] == "1")
levels(flowerN$color) <- c("1" = "white", "2" = "yellow", "3" = "pink",
                           "4" = "red", "5" = "blue")[levels(flowerN$color)]
levels(flowerN$soil)  <- c("1" = "dry", "2" = "normal", "3" = "wet")[levels(flowerN$soil)]
flowerN

## ==> example(daisy)  on how it is used
```

```
lower.to.upper.tri.inds
```
Permute Indices for Triangular Matrices

Description

Compute index vectors for extracting or reordering of lower or upper triangular matrices that are stored as contiguous vectors.

Usage

```
lower.to.upper.tri.inds(n)
upper.to.lower.tri.inds(n)
```

Arguments

n   integer larger than 1.

Value

integer vector containing a permutation of 1:N where $N = n(n-1)/2$.

See Also

[upper.tri](#), [lower.tri](#) with a related purpose.

Examples

```
m5 <- matrix(NA,5,5)
m <- m5; m[lower.tri(m)] <- upper.to.lower.tri.inds(5); m
m <- m5; m[upper.tri(m)] <- lower.to.upper.tri.inds(5); m

stopifnot(lower.to.upper.tri.inds(2) == 1,
```

```
          lower.to.upper.tri.inds(3) == 1:3,
          upper.to.lower.tri.inds(3) == 1:3,
    sort(upper.to.lower.tri.inds(5)) == 1:10,
    sort(lower.to.upper.tri.inds(6)) == 1:15)
```

---

medoids   *Compute* pam-*consistent Medoids from Clustering*

Description

Given a data matrix or dissimilarity x for say n observational units and a clustering, compute the
pam()-consistent medoids.

Usage

```
medoids(x, clustering, diss = inherits(x, "dist"), USE.NAMES = FALSE, ...)
```

Arguments

x	Either a data matrix or data frame, or dissimilarity matrix or object, see also pam.
clustering	an integer vector of length n, the number of observations, giving for each observation the number ('id') of the cluster to which it belongs. In other words, clustering has values from 1:k where k is the number of clusters, see also partition.object and cutree(), for examples where such clustering vectors are computed.
diss	see also pam.
USE.NAMES	a logical, typical false, passed to the vapply() call computing the medoids.
...	optional further argument passed to pam(xj, k=1, ...), notably metric, or variant="f_5" to use a faster algorithm, or trace.lev = k.

Value

a numeric vector of length

Author(s)

Martin Maechler, after being asked how pam() could be used instead of kmeans(), starting from a
previous clustering.

See Also

pam, kmeans. Further, cutree() and agnes (or hclust).

Examples

```
## From example(agnes):
data(votes.repub)
agn1 <- agnes(votes.repub, metric = "manhattan", stand = TRUE)
agn2 <- agnes(daisy(votes.repub), diss = TRUE, method = "complete")
agnS <- agnes(votes.repub, method = "flexible", par.method = 0.625)

for(k in 2:11) {
  print(table(cl.k <- cutree(agnS, k=k)))
  stopifnot(length(cl.k) == nrow(votes.repub), 1 <= cl.k, cl.k <= k, table(cl.k) >= 2)
  m.k <- medoids(votes.repub, cl.k)
  cat("k =", k,"; sort(medoids) = "); dput(sort(m.k), control={})
}
```

---

mona   *MONothetic Analysis Clustering of Binary Variables*

Description

Returns a list representing a divisive hierarchical clustering of a dataset with binary variables only.

Usage

```
mona(x, trace.lev = 0)
```

Arguments

x   data matrix or data frame in which each row corresponds to an observation, and
                each column corresponds to a variable. All variables must be binary. A limited
                number of missing values (NAs) is allowed. Every observation must have at least
                one value different from NA. No variable should have half of its values missing.
                There must be at least one variable which has no missing values. A variable
                with all its non-missing values identical is not allowed.

trace.lev   logical or integer indicating if (and how much) the algorithm should produce
                progress output.

Details

mona is fully described in chapter 7 of Kaufman and Rousseeuw (1990). It is "monothetic" in the
sense that each division is based on a single (well-chosen) variable, whereas most other hierarchical
methods (including agnes and diana) are "polythetic", i.e. they use all variables together.

The mona-algorithm constructs a hierarchy of clusterings, starting with one large cluster. Clusters
are divided until all observations in the same cluster have identical values for all variables.
At each stage, all clusters are divided according to the values of one variable. A cluster is divided
into one cluster with all observations having value 1 for that variable, and another cluster with all
observations having value 0 for that variable.

The variable used for splitting a cluster is the variable with the maximal total association to the other variables, according to the observations in the cluster to be splitted. The association between variables f and g is given by a(f,g)*d(f,g) - b(f,g)*c(f,g), where a(f,g), b(f,g), c(f,g), and d(f,g) are the numbers in the contingency table of f and g. [That is, a(f,g) (resp. d(f,g)) is the number of observations for which f and g both have value 0 (resp. value 1); b(f,g) (resp. c(f,g)) is the number of observations for which f has value 0 (resp. 1) and g has value 1 (resp. 0).] The total association of a variable f is the sum of its associations to all variables.

Value

an object of class ″mona″ representing the clustering. See `mona.object` for details.

Missing Values (NAs)

The mona-algorithm requires "pure" 0-1 values. However, mona(x) allows x to contain (not too many) NAs. In a preliminary step, these are "imputed", i.e., all missing values are filled in. To do this, the same measure of association between variables is used as in the algorithm. When variable f has missing values, the variable g with the largest absolute association to f is looked up. When the association between f and g is positive, any missing value of f is replaced by the value of g for the same observation. If the association between f and g is negative, then any missing value of f is replaced by the value of 1-g for the same observation.

Note

In **cluster** versions before 2.0.6, the algorithm entered an infinite loop in the boundary case of one variable, i.e., ncol(x) == 1, which currently signals an error (because the algorithm now in C, haes not correctly taken account of this special case).

See Also

`agnes` for background and references; `mona.object`, `plot.mona`.

Examples

```
data(animals)
ma <- mona(animals)
ma
## Plot similar to Figure 10 in Struyf et al (1996)
plot(ma)

## One place to see if/how error messages are *translated* (to 'de' / 'pl'):
ani.NA   <- animals; ani.NA[4,] <- NA
aniNA    <- within(animals, { end[2:9] <- NA })
aniN2    <- animals; aniN2[cbind(1:6, c(3, 1, 4:6, 2))] <- NA
ani.non2 <- within(animals, end[7] <- 3 )
ani.idNA <- within(animals, end[!is.na(end)] <- 1 )
try( mona(ani.NA)  ) ## error: .. object with all values missing
try( mona(aniNA)   ) ## error: .. more than half missing values
try( mona(aniN2)   ) ## error: all have at least one missing
try( mona(ani.non2) ) ## error: all must be binary
try( mona(ani.idNA) ) ## error:  ditto
```

---

mona.object   *Monothetic Analysis (MONA) Object*

Description

The objects of class ″mona″ represent the divisive hierarchical clustering of a dataset with only binary variables (measurements). This class of objects is returned from mona.

Value

A legitimate mona object is a list with the following components:

data	matrix with the same dimensions as the original data matrix, but with factors coded as 0 and 1, and all missing values replaced.
order	a vector giving a permutation of the original observations to allow for plotting, in the sense that the branches of a clustering tree will not cross.
order.lab	a vector similar to order, but containing observation labels instead of observation numbers. This component is only available if the original observations were labelled.
variable	vector of length n-1 where n is the number of observations, specifying the variables used to separate the observations of order.
step	vector of length n-1 where n is the number of observations, specifying the separation steps at which the observations of order are separated.

METHODS

The ″mona″ class has methods for the following generic functions: print, summary, plot.

See Also

mona for examples etc, plot.mona.

---

pam   *Partitioning Around Medoids*

Description

Partitioning (clustering) of the data into k clusters "around medoids", a more robust version of K-means.

Usage

```
pam(x, k, diss = inherits(x, "dist"),
    metric = c("euclidean", "manhattan"),
    medoids = if(is.numeric(nstart)) "random",
    nstart = if(variant == "faster") 1 else NA,
    stand = FALSE, cluster.only = FALSE,
    do.swap = TRUE,
    keep.diss = !diss && !cluster.only && n < 100,
    keep.data = !diss && !cluster.only,
    variant = c("original", "o_1", "o_2", "f_3", "f_4", "f_5", "faster"),
    pamonce = FALSE, trace.lev = 0)
```

Arguments

x	data matrix or data frame, or dissimilarity matrix or object, depending on the value of the diss argument.
	In case of a matrix or data frame, each row corresponds to an observation, and each column corresponds to a variable. All variables must be numeric (or logical). Missing values (NAs) *are* allowed—as long as every pair of observations has at least one case not missing.
	In case of a dissimilarity matrix, x is typically the output of daisy or dist. Also a vector of length n*(n-1)/2 is allowed (where n is the number of observations), and will be interpreted in the same way as the output of the above-mentioned functions. Missing values (NAs) are *not* allowed.
k	positive integer specifying the number of clusters, less than the number of observations.
diss	logical flag: if TRUE (default for dist or dissimilarity objects), then x will be considered as a dissimilarity matrix. If FALSE, then x will be considered as a matrix of observations by variables.
metric	character string specifying the metric to be used for calculating dissimilarities between observations.
	The currently available options are "euclidean" and "manhattan". Euclidean distances are root sum-of-squares of differences, and manhattan distances are the sum of absolute differences. If x is already a dissimilarity matrix, then this argument will be ignored.
medoids	NULL (default) or length-k vector of integer indices (in 1:n) specifying initial medoids instead of using the '*build*' algorithm.
nstart	used only when medoids = "random": specifies the *number* of random "starts"; this argument corresponds to the one of kmeans() (from R's package **stats**).
stand	logical; if true, the measurements in x are standardized before calculating the dissimilarities. Measurements are standardized for each variable (column), by subtracting the variable's mean value and dividing by the variable's mean absolute deviation. If x is already a dissimilarity matrix, then this argument will be ignored.
cluster.only	logical; if true, only the clustering will be computed and returned, see details.

do.swap	logical indicating if the **swap** phase should happen. The default, TRUE, correspond to the original algorithm. On the other hand, the **swap** phase is much more computer intensive than the **build** one for large n, so can be skipped by do.swap = FALSE.
keep.diss, keep.data	
	logicals indicating if the dissimilarities and/or input data x should be kept in the result. Setting these to FALSE can give much smaller results and hence even save memory allocation *time*.
pamonce	logical or integer in 0:6 specifying algorithmic short cuts as proposed by Reynolds et al. (2006), and Schubert and Rousseeuw (2019, 2021) see below.
variant	a [character](#) string denoting the variant of PAM algorithm to use; a more self-documenting version of pamonce which should be used preferably; note that "faster" not only uses pamonce = 6 but also nstart = 1 and hence medoids = "random" by default.
trace.lev	integer specifying a trace level for printing diagnostics during the build and swap phase of the algorithm. Default 0 does not print anything; higher values print increasingly more.

Details

The basic pam algorithm is fully described in chapter 2 of Kaufman and Rousseeuw(1990). Compared to the k-means approach in kmeans, the function pam has the following features: (a) it also accepts a dissimilarity matrix; (b) it is more robust because it minimizes a sum of dissimilarities instead of a sum of squared euclidean distances; (c) it provides a novel graphical display, the silhouette plot (see plot.partition) (d) it allows to select the number of clusters using mean([silhouette](#)(pr)[, "sil_width"]) on the result pr <- pam(..), or directly its component pr$silinfo$avg.width, see also [pam.object](#).

When cluster.only is true, the result is simply a (possibly named) integer vector specifying the clustering, i.e.,
pam(x,k, cluster.only=TRUE) is the same as
pam(x,k)$clustering but computed more efficiently.

The pam-algorithm is based on the search for k representative objects or medoids among the observations of the dataset. These observations should represent the structure of the data. After finding a set of k medoids, k clusters are constructed by assigning each observation to the nearest medoid. The goal is to find k representative objects which minimize the sum of the dissimilarities of the observations to their closest representative object.
By default, when medoids are not specified, the algorithm first looks for a good initial set of medoids (this is called the **build** phase). Then it finds a local minimum for the objective function, that is, a solution such that there is no single switch of an observation with a medoid (i.e. a 'swap') that will decrease the objective (this is called the **swap** phase).

When the medoids are specified (or randomly generated), their order does *not* matter; in general, the algorithms have been designed to not depend on the order of the observations.

The pamonce option, new in cluster 1.14.2 (Jan. 2012), has been proposed by Matthias Studer, University of Geneva, based on the findings by Reynolds et al. (2006) and was extended by Erich Schubert, TU Dortmund, with the FastPAM optimizations.

The default `FALSE` (or integer `0`) corresponds to the original "swap" algorithm, whereas `pamonce = 1` (or `TRUE`), corresponds to the first proposal .... and `pamonce = 2` additionally implements the second proposal as well.

The key ideas of 'FastPAM' (Schubert and Rousseeuw, 2019) are implemented except for the linear approximate build as follows:

`pamonce = 3`**:** reduces the runtime by a factor of O(k) by exploiting that points cannot be closest to all current medoids at the same time.

`pamonce = 4`**:** additionally allows executing multiple swaps per iteration, usually reducing the number of iterations.

`pamonce = 5`**:** adds minor optimizations copied from the `pamonce = 2` approach, and is expected to be the fastest of the 'FastPam' variants included.

'FasterPAM' (Schubert and Rousseeuw, 2021) is implemented via

`pamonce = 6`**:** execute each swap which improves results immediately, and hence typically multiple swaps per iteration; this swapping algorithm runs in $O(n^2)$ rather than $O(n(n-k)k)$ time which is much faster for all but small k.

In addition, 'FasterPAM' uses *random* initialization of the medoids (instead of the '*build*' phase) to avoid the $O(n^2k)$ initialization cost of the build algorithm. In particular for large k, this yields a much faster algorithm, while preserving a similar result quality.

One may decide to use *repeated* random initialization by setting nstart > 1.

Value

an object of class `"pam"` representing the clustering. See `?pam.object` for details.

Note

For large datasets, pam may need too much memory or too much computation time since both are $O(n^2)$. Then, `clara`() is preferable, see its documentation.

There is hard limit currently, $n \leq 65536$, at 2^{16} because for larger n, $n(n-1)/2$ is larger than the maximal integer (`.Machine`$integer.max = $2^{31} - 1$).

Author(s)

Kaufman and Rousseeuw's orginal Fortran code was translated to C and augmented in several ways, e.g. to allow `cluster.only=TRUE` or `do.swap=FALSE`, by Martin Maechler.
Matthias Studer, Univ.Geneva provided the `pamonce` (1 and 2) implementation.
Erich Schubert, TU Dortmund contributed the `pamonce` (3 to 6) implementation.

References

Reynolds, A., Richards, G., de la Iglesia, B. and Rayward-Smith, V. (1992) Clustering rules: A comparison of partitioning and hierarchical clustering algorithms; *Journal of Mathematical Modelling and Algorithms* **5**, 475–504. doi:10.1007/s1085200590221.

Erich Schubert and Peter J. Rousseeuw (2019) Faster k-Medoids Clustering: Improving the PAM, CLARA, and CLARANS Algorithms; SISAP 2020, 171–187. doi:10.1007/9783030320478_16.

Erich Schubert and Peter J. Rousseeuw (2021) Fast and Eager k-Medoids Clustering: O(k) Runtime Improvement of the PAM, CLARA, and CLARANS Algorithms; Preprint, to appear in Information Systems (https://arxiv.org/abs/2008.05171).

See Also

agnes for background and references; pam.object, clara, daisy, partition.object, plot.partition, dist.

Examples

```
## generate 25 objects, divided into 2 clusters.
set.seed(17) # to get reproducible data:
x <- rbind(cbind(rnorm(10,0,0.5), rnorm(10,0,0.5)),
           cbind(rnorm(15,5,0.5), rnorm(15,5,0.5)))
pamx <- pam(x, 2)
pamx # Medoids: '9' and '15' ...
summary(pamx)
plot(pamx)
stopifnot(pamx$id.med == c(9, 15))
stopifnot(identical(pamx$clustering, rep(1:2, c(10, 15))))

## use obs. 1 & 16 as starting medoids -- same result (for seed above, *and* typically) :
(p2m <- pam(x, 2, medoids = c(1,16)))
## no _build_ *and* no _swap_ phase: just cluster all obs. around (1, 16):
p2.s <- pam(x, 2, medoids = c(1,16), do.swap = FALSE)
p2.s
keep_nms <- setdiff(names(pamx), c("call", "objective"))# .$objective["build"] differ
stopifnot(p2.s$id.med == c(1,16), # of course
          identical(pamx[keep_nms],
                    p2m[keep_nms]))

p3m <- pam(x, 3, trace.lev = 2)
## rather stupid initial medoids:
(p3m. <- pam(x, 3, medoids = 3:1, trace.lev = 1))


pam(daisy(x, metric = "manhattan"), 2, diss = TRUE)

data(ruspini)
## Plot similar to Figure 4 in Stryuf et al (1996)
## Not run: plot(pam(ruspini, 4), ask = TRUE)
```

---

pam.object   *Partitioning Around Medoids (PAM) Object*

Description

The objects of class "pam" represent a partitioning of a dataset into clusters.

Value

A legitimate pam object is a [list](#) with the following components:

medoids   the medoids or representative objects of the clusters. If a dissimilarity matrix
                was given as input to pam, then a vector of numbers or labels of observations is
                given, else medoids is a [matrix](#) with in each row the coordinates of one medoid.

id.med   integer vector of *indices* giving the medoid observation numbers.

clustering   the clustering vector, see [partition.object](#).

objective   the objective function after the first and second step of the pam algorithm.

isolation   vector with length equal to the number of clusters, specifying which clusters are
                isolated clusters (L- or L*-clusters) and which clusters are not isolated.
                A cluster is an L*-cluster iff its diameter is smaller than its separation. A cluster
                is an L-cluster iff for each observation i the maximal dissimilarity between i and
                any other observation of the cluster is smaller than the minimal dissimilarity
                between i and any observation of another cluster. Clearly each L*-cluster is also
                an L-cluster.

clusinfo   matrix, each row gives numerical information for one cluster. These are the
                cardinality of the cluster (number of observations), the maximal and average
                dissimilarity between the observations in the cluster and the cluster's medoid,
                the diameter of the cluster (maximal dissimilarity between two observations of
                the cluster), and the separation of the cluster (minimal dissimilarity between an
                observation of the cluster and an observation of another cluster).

silinfo   list with silhouette width information, see [partition.object](#).

diss   dissimilarity (maybe NULL), see [partition.object](#).

call   generating call, see [partition.object](#).

data   (possibibly standardized) see [partition.object](#).

GENERATION

These objects are returned from [pam](#).

METHODS

The "pam" class has methods for the following generic functions: print, summary.

INHERITANCE

The class "pam" inherits from "partition". Therefore, the generic functions plot and clusplot
can be used on a pam object.

See Also

[pam](#), [dissimilarity.object](#), [partition.object](#), [plot.partition](#).

Examples

```
## Use the silhouette widths for assessing the best number of clusters,
## following a one-dimensional example from Christian Hennig :
##
x <- c(rnorm(50), rnorm(50,mean=5), rnorm(30,mean=15))
asw <- numeric(20)
## Note that "k=1" won't work!
for (k in 2:20)
  asw[k] <- pam(x, k) $ silinfo $ avg.width
k.best <- which.max(asw)
cat("silhouette-optimal number of clusters:", k.best, "\n")

plot(1:20, asw, type= "h", main = "pam() clustering assessment",
     xlab= "k  (# clusters)", ylab = "average silhouette width")
axis(1, k.best, paste("best",k.best,sep="\n"), col = "red", col.axis = "red")
```

---

partition.object   *Partitioning Object*

Description

The objects of class "partition" represent a partitioning of a dataset into clusters.

Value

a "partition" object is a list with the following (and typically more) components:

clustering   the clustering vector. An integer vector of length n, the number of observations, giving for each observation the number ('id') of the cluster to which it belongs.

call   the matched call generating the object.

silinfo   a list with all *silhouette* information, only available when the number of clusters is non-trivial, i.e., $1 < k < n$ and then has the following components, see silhouette

> **widths** an (n x 3) matrix, as returned by silhouette(), with for each observation i the cluster to which i belongs, as well as the neighbor cluster of i (the cluster, not containing i, for which the average dissimilarity between its observations and i is minimal), and the silhouette width $s(i)$ of the observation.
>
> **clus.avg.widths** the average silhouette width per cluster.
>
> **avg.width** the average silhouette width for the dataset, i.e., simply the average of $s(i)$ over all observations i.
>
> This information is also needed to construct a *silhouette plot* of the clustering, see plot.partition.
>
> Note that avg.width can be maximized over different clusterings (e.g. with varying number of clusters) to choose an *optimal* clustering.

objective	value of criterion maximized during the partitioning algorithm, may more than one entry for different stages.
diss	an object of class `"dissimilarity"`, representing the total dissimilarity matrix of the dataset (or relevant subset, e.g. for `clara`).
data	a matrix containing the original or standardized data. This might be missing to save memory or when a dissimilarity matrix was given as input structure to the clustering method.

GENERATION

These objects are returned from `pam`, `clara` or `fanny`.

METHODS

The `"partition"` class has a method for the following generic functions: `plot`, `clusplot`.

INHERITANCE

The following classes inherit from class `"partition"` : `"pam"`, `"clara"` and `"fanny"`.

See `pam.object`, `clara.object` and `fanny.object` for details.

See Also

`pam`, `clara`, `fanny`.

---

plantTraits   *Plant Species Traits Data*

Description

This dataset constitutes a description of 136 plant species according to biological attributes (morphological or reproductive)

Usage

```
data(plantTraits)
```

Format

A data frame with 136 observations on the following 31 variables.

pdias Diaspore mass (mg)

longindex Seed bank longevity

durflow Flowering duration

height Plant height, an ordered factor with levels $1 < 2 < \ldots < 8$.

begflow Time of first flowering, an ordered factor with levels $1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9$

`mycor` Mycorrhizas, an ordered factor with levels 0never < 1 sometimes< 2always

`vegaer` aerial vegetative propagation, an ordered factor with levels 0never < 1 present but limited< 2important.

`vegsout` underground vegetative propagation, an ordered factor with 3 levels identical to `vegaer` above.

`autopoll` selfing pollination, an ordered factor with levels 0never < 1rare < 2 often< the rule3

`insects` insect pollination, an ordered factor with 5 levels $0 < \ldots < 4$.

`wind` wind pollination, an ordered factor with 5 levels $0 < \ldots < 4$.

`lign` a binary factor with levels 0:1, indicating if plant is woody.

`piq` a binary factor indicating if plant is thorny.

`ros` a binary factor indicating if plant is rosette.

`semiros` semi-rosette plant, a binary factor (0: no; 1: yes).

`leafy` leafy plant, a binary factor.

`suman` summer annual, a binary factor.

`winan` winter annual, a binary factor.

`monocarp` monocarpic perennial, a binary factor.

`polycarp` polycarpic perennial, a binary factor.

`seasaes` seasonal aestival leaves, a binary factor.

`seashiv` seasonal hibernal leaves, a binary factor.

`seasver` seasonal vernal leaves, a binary factor.

`everalw` leaves always evergreen, a binary factor.

`everparti` leaves partially evergreen, a binary factor.

`elaio` fruits with an elaiosome (dispersed by ants), a binary factor.

`endozoo` endozoochorous fruits, a binary factor.

`epizoo` epizoochorous fruits, a binary factor.

`aquat` aquatic dispersal fruits, a binary factor.

`windgl` wind dispersed fruits, a binary factor.

`unsp` unspecialized mechanism of seed dispersal, a binary factor.

Details

Most of factor attributes are not disjunctive. For example, a plant can be usually pollinated by insects but sometimes self-pollination can occured.

Source

Vallet, Jeanne (2005) *Structuration de communautés végétales et analyse comparative de traits biologiques le long d'un gradient d'urbanisation*. Mémoire de Master 2 'Ecologie-Biodiversité-Evolution'; Université Paris Sud XI, 30p.+ annexes (in french)

Examples

```
data(plantTraits)

## Calculation of a dissimilarity matrix
library(cluster)
dai.b <- daisy(plantTraits,
               type = list(ordratio = 4:11, symm = 12:13, asymm = 14:31))

## Hierarchical classification
agn.trts <- agnes(dai.b, method="ward")
plot(agn.trts, which.plots = 2, cex= 0.6)
plot(agn.trts, which.plots = 1)
cutree6 <- cutree(agn.trts, k=6)
cutree6

## Principal Coordinate Analysis
cmdsdai.b <- cmdscale(dai.b, k=6)
plot(cmdsdai.b[, 1:2], asp = 1, col = cutree6)
```

---

plot.agnes   *Plots of an Agglomerative Hierarchical Clustering*

Description

Creates plots for visualizing an agnes object.

Usage

```
## S3 method for class 'agnes'
plot(x, ask = FALSE, which.plots = NULL, main = NULL,
           sub = paste("Agglomerative Coefficient = ",round(x$ac, digits = 2)),
           adj = 0, nmax.lab = 35, max.strlen = 5, xax.pretty = TRUE, ...)
```

Arguments

x	an object of class "agnes", typically created by agnes(.).
ask	logical; if true and which.plots is NULL, plot.agnes operates in interactive mode, via menu.
which.plots	integer vector or NULL (default), the latter producing both plots. Otherwise, which.plots must contain integers of 1 for a *banner* plot or 2 for a dendrogram or "clustering tree".
main, sub	main and sub title for the plot, with convenient defaults. See documentation for these arguments in plot.default.
adj	for label adjustment in bannerplot().
nmax.lab	integer indicating the number of labels which is considered too large for single-name labelling the banner plot.

max.strlen	positive integer giving the length to which strings are truncated in banner plot labeling.
xax.pretty	logical or integer indicating if pretty(*, n = xax.pretty) should be used for the x axis. xax.pretty = FALSE is for back compatibility.
...	graphical parameters (see par) may also be supplied and are passed to bannerplot() or pltree() (see pltree.twins), respectively.

Details

When ask = TRUE, rather than producing each plot sequentially, plot.agnes displays a menu listing all the plots that can be produced. If the menu is not desired but a pause between plots is still wanted one must set par(ask= TRUE) before invoking the plot command.

The banner displays the hierarchy of clusters, and is equivalent to a tree. See Rousseeuw (1986) or chapter 5 of Kaufman and Rousseeuw (1990). The banner plots distances at which observations and clusters are merged. The observations are listed in the order found by the agnes algorithm, and the numbers in the height vector are represented as bars between the observations.

The leaves of the clustering tree are the original observations. Two branches come together at the distance between the two clusters being merged.

For more customization of the plots, rather call bannerplot and pltree(), i.e., its method pltree.twins, respectively.

directly with corresponding arguments, e.g., xlab or ylab.

Side Effects

Appropriate plots are produced on the current graphics device. This can be one or both of the following choices:
Banner
Clustering tree

Note

In the banner plot, observation labels are only printed when the number of observations is limited less than nmax.lab (35, by default), for readability. Moreover, observation labels are truncated to maximally max.strlen (5) characters.

For the dendrogram, more flexibility than via pltree() is provided by dg <- as.dendrogram(x) and plotting dg via plot.dendrogram.

References

Kaufman, L. and Rousseeuw, P.J. (1990) *Finding Groups in Data: An Introduction to Cluster Analysis*. Wiley, New York.

Rousseeuw, P.J. (1986). A visual display for hierarchical classification, in *Data Analysis and Informatics 4*; edited by E. Diday, Y. Escoufier, L. Lebart, J. Pages, Y. Schektman, and R. Tomassone. North-Holland, Amsterdam, 743–748.

Struyf, A., Hubert, M. and Rousseeuw, P.J. (1997) Integrating Robust Clustering Techniques in S-PLUS, *Computational Statistics and Data Analysis*, **26**, 17–37.

See Also

agnes and agnes.object; bannerplot, pltree.twins, and par.

Examples

```
## Can also pass 'labels' to pltree() and bannerplot():
data(iris)
cS <- as.character(Sp <- iris$Species)
cS[Sp == "setosa"] <- "S"
cS[Sp == "versicolor"] <- "V"
cS[Sp == "virginica"] <- "g"
ai <- agnes(iris[, 1:4])
plot(ai, labels = cS, nmax.lab = 150)# bannerplot labels are mess
```

---

plot.diana   *Plots of a Divisive Hierarchical Clustering*

Description

Creates plots for visualizing a diana object.

Usage

```
## S3 method for class 'diana'
plot(x, ask = FALSE, which.plots = NULL, main = NULL,
            sub = paste("Divisive Coefficient = ", round(x$dc, digits = 2)),
            adj = 0, nmax.lab = 35, max.strlen = 5, xax.pretty = TRUE, ...)
```

Arguments

x	an object of class "diana", typically created by diana(.).
ask	logical; if true and which.plots is NULL, plot.diana operates in interactive mode, via menu.
which.plots	integer vector or NULL (default), the latter producing both plots. Otherwise, which.plots must contain integers of 1 for a *banner* plot or 2 for a dendrogram or "clustering tree".
main, sub	main and sub title for the plot, each with a convenient default. See documentation for these arguments in plot.default.
adj	for label adjustment in bannerplot().
nmax.lab	integer indicating the number of labels which is considered too large for single-name labelling the banner plot.
max.strlen	positive integer giving the length to which strings are truncated in banner plot labeling.
xax.pretty	logical or integer indicating if pretty(*, n = xax.pretty) should be used for the x axis. xax.pretty = FALSE is for back compatibility.
...	graphical parameters (see par) may also be supplied and are passed to bannerplot() or pltree(), respectively.

Details

When ask = TRUE, rather than producing each plot sequentially, plot.diana displays a menu listing all the plots that can be produced. If the menu is not desired but a pause between plots is still wanted one must set par(ask= TRUE) before invoking the plot command.

The banner displays the hierarchy of clusters, and is equivalent to a tree. See Rousseeuw (1986) or chapter 6 of Kaufman and Rousseeuw (1990). The banner plots the diameter of each cluster being splitted. The observations are listed in the order found by the diana algorithm, and the numbers in the height vector are represented as bars between the observations.

The leaves of the clustering tree are the original observations. A branch splits up at the diameter of the cluster being splitted.

Side Effects

An appropriate plot is produced on the current graphics device. This can be one or both of the following choices:
Banner
Clustering tree

Note

In the banner plot, observation labels are only printed when the number of observations is limited less than nmax.lab (35, by default), for readability. Moreover, observation labels are truncated to maximally max.strlen (5) characters.

References

see those in `plot.agnes`.

See Also

`diana`, `diana.object`, `twins.object`, `par`.

Examples

```
example(diana)# -> dv <- diana(....)

plot(dv, which.plots = 1, nmax.lab = 100)

## wider labels :
op <- par(mar = par("mar") + c(0, 2, 0,0))
plot(dv, which.plots = 1, nmax.lab = 100, max.strlen = 12)
par(op)
```

---

plot.mona   *Banner of Monothetic Divisive Hierarchical Clusterings*

Description

Creates the banner of a mona object.

Usage

```
## S3 method for class 'mona'
plot(x, main = paste("Banner of ", deparse1(x$call)),
          sub = NULL, xlab = "Separation step",
  col = c(2,0), axes = TRUE, adj = 0,
          nmax.lab = 35, max.strlen = 5, ...)
```

Arguments

x	an object of class "mona", typically created by mona(.).
main, sub	main and sub titles for the plot, with convenient defaults. See documentation in plot.default.
xlab	x axis label, see title.
col, adj	graphical parameters passed to bannerplot().
axes	logical, indicating if (labeled) axes should be drawn.
nmax.lab	integer indicating the number of labels which is considered too large for labeling.
max.strlen	positive integer giving the length to which strings are truncated in labeling.
...	further graphical arguments are passed to bannerplot() and text.

Details

Plots the separation step at which clusters are splitted. The observations are given in the order found by the mona algorithm, the numbers in the step vector are represented as bars between the observations.

When a long bar is drawn between two observations, those observations have the same value for each variable. See chapter 7 of Kaufman and Rousseeuw (1990).

Side Effects

A banner is plotted on the current graphics device.

Note

In the banner plot, observation labels are only printed when the number of observations is limited less than nmax.lab (35, by default), for readability. Moreover, observation labels are truncated to maximally max.strlen (5) characters.

References

see those in `plot.agnes`.

See Also

`mona`, `mona.object`, `par`.

`plot.partition`   *Plot of a Partition of the Data Set*

Description

Creates plots for visualizing a `partition` object.

Usage

```
## S3 method for class 'partition'
plot(x, ask = FALSE, which.plots = NULL,
     nmax.lab = 40, max.strlen = 5, data = x$data, dist = NULL,
     stand = FALSE, lines = 2,
     shade = FALSE, color = FALSE, labels = 0, plotchar = TRUE,
     span = TRUE, xlim = NULL, ylim = NULL, main = NULL, ...)
```

Arguments

x	an object of class "partition", typically created by the functions `pam`, `clara`, or `fanny`.
ask	logical; if true and `which.plots` is NULL, `plot.partition` operates in interactive mode, via `menu`.
which.plots	integer vector or NULL (default), the latter producing both plots. Otherwise, `which.plots` must contain integers of 1 for a *clusplot* or 2 for *silhouette*.
nmax.lab	integer indicating the number of labels which is considered too large for single-name labeling the silhouette plot.
max.strlen	positive integer giving the length to which strings are truncated in silhouette plot labeling.
data	numeric matrix with the scaled data; per default taken from the partition object x, but can be specified explicitly.
dist	when x does not have a `diss` component as for `pam`(*, keep.diss=FALSE), `dist` must be the dissimilarity if a clusplot is desired.
stand, lines, shade, color, labels, plotchar, span, xlim, ylim, main, ...	
	All optional arguments available for the `clusplot.default` function (except for the `diss` one) and graphical parameters (see `par`) may also be supplied as arguments to this function.

Details

When ask= TRUE, rather than producing each plot sequentially, plot.partition displays a menu listing all the plots that can be produced. If the menu is not desired but a pause between plots is still wanted, call par(ask= TRUE) before invoking the plot command.

The *clusplot* of a cluster partition consists of a two-dimensional representation of the observations, in which the clusters are indicated by ellipses (see clusplot.partition for more details).

The *silhouette plot* of a nonhierarchical clustering is fully described in Rousseeuw (1987) and in chapter 2 of Kaufman and Rousseeuw (1990). For each observation i, a bar is drawn, representing its silhouette width s(i), see silhouette for details. Observations are grouped per cluster, starting with cluster 1 at the top. Observations with a large s(i) (almost 1) are very well clustered, a small s(i) (around 0) means that the observation lies between two clusters, and observations with a negative s(i) are probably placed in the wrong cluster.

A clustering can be performed for several values of k (the number of clusters). Finally, choose the value of k with the largest overall average silhouette width.

Side Effects

An appropriate plot is produced on the current graphics device. This can be one or both of the following choices:
Clusplot
Silhouette plot

Note

In the silhouette plot, observation labels are only printed when the number of observations is less than nmax.lab (40, by default), for readability. Moreover, observation labels are truncated to maximally max.strlen (5) characters.
For more flexibility, use plot(silhouette(x), ...), see plot.silhouette.

References

Rousseeuw, P.J. (1987) Silhouettes: A graphical aid to the interpretation and validation of cluster analysis. *J. Comput. Appl. Math.*, **20**, 53–65.

Further, the references in plot.agnes.

See Also

partition.object, clusplot.partition, clusplot.default, pam, pam.object, clara, clara.object, fanny, fanny.object, par.

Examples

```
## generate 25 objects, divided into 2 clusters.
x <- rbind(cbind(rnorm(10,0,0.5), rnorm(10,0,0.5)),
           cbind(rnorm(15,5,0.5), rnorm(15,5,0.5)))
plot(pam(x, 2))

## Save space not keeping data in clus.object, and still clusplot() it:
```

```
data(xclara)
cx <- clara(xclara, 3, keep.data = FALSE)
cx$data # is NULL
plot(cx, data = xclara)
```

---

pltree   *Plot Clustering Tree of a Hierarchical Clustering*

Description

pltree() Draws a clustering tree ("dendrogram") on the current graphics device. We provide the twins method draws the tree of a twins object, i.e., hierarchical clustering, typically resulting from [agnes](../pltree)() or [diana](../pltree)().

Usage

```
pltree(x, ...)
## S3 method for class 'twins'
pltree(x, main = paste("Dendrogram of ", deparse1(x$call)),
              labels = NULL, ylab = "Height", ...)
```

Arguments

x	in general, an R object for which a pltree method is defined; specifically, an object of class "twins", typically created by either [agnes](../pltree)() or [diana](../pltree)().
main	main title with a sensible default.
labels	labels to use; the default is constructed from x.
ylab	label for y-axis.
...	graphical parameters (see [par](../par)) may also be supplied as arguments to this function.

Details

Creates a plot of a clustering tree given a twins object. The leaves of the tree are the original observations. In case of an agglomerative clustering, two branches come together at the distance between the two clusters being merged. For a divisive clustering, a branch splits up at the diameter of the cluster being splitted.

Note that currently the method function simply calls plot([as.hclust](../as.hclust)(x), ...), which dispatches to [plot.hclust](../plot.hclust)(..). If more flexible plots are needed, consider xx <- [as.dendrogram](../as.dendrogram)([as.hclust](../as.hclust)(x)) and plotting xx, see [plot.dendrogram](../plot.dendrogram).

Value

a NULL value is returned.

See Also

agnes, agnes.object, diana, diana.object, hclust, par, plot.agnes, plot.diana.

Examples

```
data(votes.repub)
agn <- agnes(votes.repub)
pltree(agn)

dagn  <- as.dendrogram(as.hclust(agn))
dagn2 <- as.dendrogram(as.hclust(agn), hang = 0.2)
op <- par(mar = par("mar") + c(0,0,0, 2)) # more space to the right
plot(dagn2, horiz = TRUE)
plot(dagn, horiz = TRUE, center = TRUE,
     nodePar = list(lab.cex = 0.6, lab.col = "forest green", pch = NA),
     main = deparse(agn$call))
par(op)
```

---

pluton   *Isotopic Composition Plutonium Batches*

Description

The pluton data frame has 45 rows and 4 columns, containing percentages of isotopic composition of 45 Plutonium batches.

Usage

```
data(pluton)
```

Format

This data frame contains the following columns:

Pu238 the percentages of ^{238}Pu, always less than 2 percent.

Pu239 the percentages of ^{239}Pu, typically between 60 and 80 percent (from neutron capture of Uranium, ^{238}U).

Pu240 percentage of the plutonium 240 isotope.

Pu241 percentage of the plutonium 241 isotope.

Details

Note that the percentage of plutonium~242 can be computed from the other four percentages, see the examples.

In the reference below it is explained why it is very desirable to combine these plutonium patches in three groups of similar size.

Source

Available as 'pluton.dat' from the archive of the University of Antwerpen, '..../datasets/clusplot-examples.tar.gz
no longer available.

References

Rousseeuw, P.J. and Kaufman, L and Trauwaert, E. (1996) Fuzzy clustering using scatter matrices,
Computational Statistics and Data Analysis **23**(1), 135–151.

Examples

```
data(pluton)

hist(apply(pluton,1,sum), col = "gray") # between 94% and 100%
pu5 <- pluton
pu5$Pu242 <- 100 - apply(pluton,1,sum) # the remaining isotope.
pairs(pu5)
```

---

predict.ellipsoid *Predict Method for Ellipsoid Objects*

Description

Compute points on the ellipsoid boundary, mostly for drawing.

Usage

```
predict.ellipsoid(object, n.out=201, ...)
## S3 method for class 'ellipsoid'
predict(object, n.out=201, ...)
ellipsoidPoints(A, d2, loc, n.half = 201)
```

Arguments

object	an object of class ellipsoid, typically from [ellipsoidhull](); alternatively any list-like object with proper components, see details below.
n.out, n.half	half the number of points to create.
A, d2, loc	arguments of the auxilary ellipsoidPoints, see below.
...	passed to and from methods.

Details

Note ellipsoidPoints is the workhorse function of predict.ellipsoid a standalone function
and method for ellipsoid objects, see [ellipsoidhull](). The class of object is not checked; it
must solely have valid components loc (length p), the $p \times p$ matrix cov (corresponding to A) and
d2 for the center, the shape ("covariance") matrix and the squared average radius (or distance) or
qchisq(*, p) quantile.

Unfortunately, this is only implemented for $p = 2$, currently; contributions for $p \geq 3$ are *very
welcome*.

Value

a numeric matrix of dimension 2*n.out times p.

See Also

ellipsoidhull, volume.ellipsoid.

Examples

```
 ## see also  example(ellipsoidhull)

## Robust vs. L.S. covariance matrix
set.seed(143)
x <- rt(200, df=3)
y <- 3*x + rt(200, df=2)
plot(x,y, main="non-normal data (N=200)")
mtext("with classical and robust cov.matrix ellipsoids")
X <- cbind(x,y)
C.ls <- cov(X) ; m.ls <- colMeans(X)
d2.99 <- qchisq(0.99, df = 2)
lines(ellipsoidPoints(C.ls, d2.99, loc=m.ls), col="green")
if(require(MASS)) {
  Cxy <- cov.rob(cbind(x,y))
  lines(ellipsoidPoints(Cxy$cov, d2 = d2.99, loc=Cxy$center), col="red")
}# MASS
```

---

print.agnes   *Print Method for AGNES Objects*

Description

Prints the call, agglomerative coefficient, ordering of objects and distances between merging clusters ('Height') of an agnes object.

This is a method for the generic print() function for objects inheriting from class agnes, see agnes.object.

Usage

```
## S3 method for class 'agnes'
print(x, ...)
```

Arguments

x   an agnes object.

...   potential further arguments (required by generic).

See Also

summary.agnes producing more output; agnes, agnes.object, print, print.default.

---

print.clara *Print Method for CLARA Objects*

Description

Prints the best sample, medoids, clustering vector and objective function of `clara` object.

This is a method for the function [print](#)() for objects inheriting from class [clara](#).

Usage

```
## S3 method for class 'clara'
print(x, ...)
```

Arguments

x	a clara object.
...	potential further arguments (require by generic).

See Also

[summary.clara](#) producing more output; [clara](#), [clara.object](#), [print](#), [print.default](#).

---

print.diana *Print Method for DIANA Objects*

Description

Prints the ordering of objects, diameters of splitted clusters, and divisive coefficient of a `diana` object.

This is a method for the function [print](#)() for objects inheriting from class [diana](#).

Usage

```
## S3 method for class 'diana'
print(x, ...)
```

Arguments

x	a diana object.
...	potential further arguments (require by generic).

See Also

[diana](#), [diana.object](#), [print](#), [print.default](#).

---

print.dissimilarity   *Print and Summary Methods for Dissimilarity Objects*

Description

Print or summarize the distances and the attributes of a dissimilarity object.

These are methods for the functions print() and summary() for dissimilarity objects. See print, print.default, or summary for the general behavior of these.

Usage

```
## S3 method for class 'dissimilarity'
print(x, diag = NULL, upper = NULL,
      digits = getOption("digits"), justify = "none", right = TRUE, ...)
## S3 method for class 'dissimilarity'
summary(object,
        digits = max(3, getOption("digits") - 2), ...)
## S3 method for class 'summary.dissimilarity'
print(x, ...)
```

Arguments

x, object   a dissimilarity object or a summary.dissimilarity one for print.summary.dissimilarity().

digits   the number of digits to use, see print.default.

diag, upper, justify, right

                   optional arguments specifying how the triangular dissimilarity matrix is printed;
                   see print.dist.

...   potential further arguments (require by generic).

See Also

daisy, dissimilarity.object, print, print.default, print.dist.

Examples

```
## See   example(daisy)

sd <- summary(daisy(matrix(rnorm(100), 20,5)))
sd # -> print.summary.dissimilarity(.)
str(sd)
```

---

print.fanny   *Print and Summary Methods for FANNY Objects*

Description

Prints the objective function, membership coefficients and clustering vector of `fanny` object.

This is a method for the function [print](#)() for objects inheriting from class [fanny](#).

Usage

```
## S3 method for class 'fanny'
print(x, digits = getOption("digits"), ...)
## S3 method for class 'fanny'
summary(object, ...)
## S3 method for class 'summary.fanny'
print(x, digits = getOption("digits"), ...)
```

Arguments

x, object	a [fanny](#) object.
digits	number of significant digits for printing, see [print.default](#).
...	potential further arguments (required by generic).

See Also

[fanny](#), [fanny.object](#), [print](#), [print.default](#).

---

print.mona   *Print Method for MONA Objects*

Description

Prints the ordering of objects, separation steps, and used variables of a `mona` object.

This is a method for the function [print](#)() for objects inheriting from class [mona](#).

Usage

```
## S3 method for class 'mona'
print(x, ...)
```

Arguments

x	a mona object.
...	potential further arguments (require by generic).

See Also

[mona](), [mona.object](), [print](), [print.default]().

---

print.pam   *Print Method for PAM Objects*

Description

Prints the medoids, clustering vector and objective function of pam object.

This is a method for the function [print]() for objects inheriting from class [pam]().

Usage

```
## S3 method for class 'pam'
print(x, ...)
```

Arguments

x   a pam object.

...   potential further arguments (require by generic).

See Also

[pam](), [pam.object](), [print](), [print.default]().

---

ruspini   *Ruspini Data*

Description

The Ruspini data set, consisting of 75 points in four groups that is popular for illustrating clustering techniques.

Usage

```
data(ruspini)
```

Format

A data frame with 75 observations on 2 variables giving the x and y coordinates of the points, respectively.

Source

E. H. Ruspini (1970) Numerical methods for fuzzy clustering. *Inform. Sci.* **2**, 319–350.

References

see those in [agnes](#).

Examples

```
data(ruspini)

## Plot similar to Figure 4 in Stryuf et al (1996)
## Not run: plot(pam(ruspini, 4), ask = TRUE)


## Plot similar to Figure 6 in Stryuf et al (1996)
plot(fanny(ruspini, 5))
```

---

silhouette   *Compute or Extract Silhouette Information from Clustering*

Description

Compute silhouette information according to a given clustering in k clusters.

Usage

```
silhouette(x, ...)
## Default S3 method:
  silhouette(x, dist, dmatrix, ...)
## S3 method for class 'partition'
silhouette(x, ...)
## S3 method for class 'clara'
silhouette(x, full = FALSE, subset = NULL, ...)

sortSilhouette(object, ...)
## S3 method for class 'silhouette'
summary(object, FUN = mean, ...)
## S3 method for class 'silhouette'
plot(x, nmax.lab = 40, max.strlen = 5,
     main = NULL, sub = NULL, xlab = expression("Silhouette width "* s[i]),
     col = "gray",  do.col.sort = length(col) > 1, border = 0,
     cex.names = par("cex.axis"), do.n.k = TRUE, do.clus.stat = TRUE, ...)
```

Arguments

x
: an object of appropriate class; for the default method an integer vector with k different integer cluster codes or a list with such an x$clustering component. Note that silhouette statistics are only defined if $2 \le k \le n - 1$.

dist
: a dissimilarity object inheriting from class [dist](#) or coercible to one. If not specified, dmatrix must be.

dmatrix	a symmetric dissimilarity matrix ($n \times n$), specified instead of dist, which can be more efficient.
full	logical or number in $[0, 1]$ specifying if a *full* silhouette should be computed for clara object. When a number, say f, for a random sample.int(n, size = f*n) of the data the silhouette values are computed. This requires $O((f * n)^2)$ memory, since the full dissimilarity of the (sub)sample (see daisy) is needed internally.
subset	a subset from 1:n, specified instead of full to specify the indices of the observations to be used for the silhouette computations.
object	an object of class silhouette.
...	further arguments passed to and from methods.
FUN	function used to summarize silhouette widths.
nmax.lab	integer indicating the number of labels which is considered too large for single-name labeling the silhouette plot.
max.strlen	positive integer giving the length to which strings are truncated in silhouette plot labeling.
main, sub, xlab	arguments to title; have a sensible non-NULL default here.
col, border, cex.names	
	arguments passed barplot(); note that the default used to be col = heat.colors(n), border = par("fg") instead. col can also be a color vector of length k for clusterwise coloring, see also do.col.sort:
do.col.sort	logical indicating if the colors col should be sorted "along" the silhouette; this is useful for casewise or clusterwise coloring.
do.n.k	logical indicating if n and k "title text" should be written.
do.clus.stat	logical indicating if cluster size and averages should be written right to the silhouettes.

Details

For each observation i, the *silhouette width* $s(i)$ is defined as follows:
Put a(i) = average dissimilarity between i and all other points of the cluster to which i belongs (if i is the *only* observation in its cluster, $s(i) := 0$ without further calculations). For all *other* clusters C, put $d(i, C)$ = average dissimilarity of i to all observations of C. The smallest of these $d(i, C)$ is $b(i) := \min_C d(i, C)$, and can be seen as the dissimilarity between i and its "neighbor" cluster, i.e., the nearest one to which it does *not* belong. Finally,

$$s(i) := \frac{b(i) - a(i)}{max(a(i), b(i))}.$$

silhouette.default() is now based on C code donated by Romain Francois (the R version being still available as cluster:::silhouetteR).

Observations with a large $s(i)$ (almost 1) are very well clustered, a small $s(i)$ (around 0) means that the observation lies between two clusters, and observations with a negative $s(i)$ are probably placed in the wrong cluster.

Value

silhouette() returns an object, sil, of class silhouette which is an $n \times 3$ matrix with attributes. For each observation i, sil[i,] contains the cluster to which i belongs as well as the neighbor cluster of i (the cluster, not containing i, for which the average dissimilarity between its observations and i is minimal), and the silhouette width $s(i)$ of the observation. The [colnames](#) correspondingly are c("cluster", "neighbor", "sil_width").

summary(sil) returns an object of class summary.silhouette, a list with components

si.summary: numerical [summary](#) of the individual silhouette widths $s(i)$.

clus.avg.widths: numeric (rank 1) array of clusterwise *means* of silhouette widths where mean = FUN is used.

avg.width: the total mean FUN(s) where s are the individual silhouette widths.

clus.sizes: [table](#) of the k cluster sizes.

call: if available, the [call](#) creating sil.

Ordered: logical identical to attr(sil, "Ordered"), see below.

sortSilhouette(sil) orders the rows of sil as in the silhouette plot, by cluster (increasingly) and decreasing silhouette width $s(i)$.
attr(sil, "Ordered") is a logical indicating if sil *is* ordered as by sortSilhouette(). In that case, rownames(sil) will contain case labels or numbers, and
attr(sil, "iOrd") the ordering index vector.

Note

While silhouette() is *intrinsic* to the [partition](#) clusterings, and hence has a (trivial) method for these, it is straightforward to get silhouettes from hierarchical clusterings from silhouette.default() with [cutree](#)() and distance as input.

By default, for [clara](#)() partitions, the silhouette is just for the best random *subset* used. Use full = TRUE to compute (and later possibly plot) the full silhouette.

References

Rousseeuw, P.J. (1987) Silhouettes: A graphical aid to the interpretation and validation of cluster analysis. *J. Comput. Appl. Math.*, **20**, 53–65.

chapter 2 of Kaufman and Rousseeuw (1990), see the references in [plot.agnes](#).

See Also

[partition.object](#), [plot.partition](#).

Examples

```
data(ruspini)
pr4 <- pam(ruspini, 4)
str(si <- silhouette(pr4))
(ssi <- summary(si))
plot(si) # silhouette plot
```

```
plot(si, col = c("red", "green", "blue", "purple"))# with cluster-wise coloring

si2 <- silhouette(pr4$clustering, dist(ruspini, "canberra"))
summary(si2) # has small values: "canberra"'s fault
plot(si2, nmax= 80, cex.names=0.6)

op <- par(mfrow= c(3,2), oma= c(0,0, 3, 0),
          mgp= c(1.6,.8,0), mar= .1+c(4,2,2,2))
for(k in 2:6)
   plot(silhouette(pam(ruspini, k=k)), main = paste("k = ",k), do.n.k=FALSE)
mtext("PAM(Ruspini) as in Kaufman & Rousseeuw, p.101",
      outer = TRUE, font = par("font.main"), cex = par("cex.main")); frame()

## the same with cluster-wise colours:
c6 <- c("tomato", "forest green", "dark blue", "purple2", "goldenrod4", "gray20")
for(k in 2:6)
   plot(silhouette(pam(ruspini, k=k)), main = paste("k = ",k), do.n.k=FALSE,
        col = c6[1:k])
par(op)

## clara(): standard silhouette is just for the best random subset
data(xclara)
set.seed(7)
str(xc1k <- xclara[ sample(nrow(xclara), size = 1000) ,]) # rownames == indices
cl3 <- clara(xc1k, 3)
plot(silhouette(cl3))# only of the "best" subset of 46
## The full silhouette: internally needs large (36 MB) dist object:
sf <- silhouette(cl3, full = TRUE) ## this is the same as
s.full <- silhouette(cl3$clustering, daisy(xc1k))
stopifnot(all.equal(sf, s.full, check.attributes = FALSE, tolerance = 0))
## color dependent on original "3 groups of each 1000": % __FIXME ??__
plot(sf, col = 2+ as.integer(names(cl3$clustering) ) %/% 1000,
     main ="plot(silhouette(clara(.), full = TRUE))")

## Silhouette for a hierarchical clustering:
ar <- agnes(ruspini)
si3 <- silhouette(cutree(ar, k = 5), # k = 4 gave the same as pam() above
                  daisy(ruspini))
stopifnot(is.data.frame(di3 <- as.data.frame(si3)))
plot(si3, nmax = 80, cex.names = 0.5)
## 2 groups: Agnes() wasn't too good:
si4 <- silhouette(cutree(ar, k = 2), daisy(ruspini))
plot(si4, nmax = 80, cex.names = 0.5)
```

---

sizeDiss   *Sample Size of Dissimilarity Like Object*

Description

Returns the number of observations (*sample size*) corresponding to a dissimilarity like object, or equivalently, the number of rows or columns of a matrix when only the lower or upper triangular

part (without diagonal) is given.

It is nothing else but the inverse function of $f(n) = n(n-1)/2$.

Usage

```
sizeDiss(d)
```

Arguments

d   any R object with length (typically) $n(n-1)/2$.

Value

a number; n if length(d) == n(n-1)/2, NA otherwise.

See Also

[dissimilarity.object](#) and also [as.dist](#) for class dissimilarity and dist objects which have a Size attribute.

Examples

```
sizeDiss(1:10)# 5, since 10 == 5 * (5 - 1) / 2
sizeDiss(1:9) # NA

n <- 1:100
stopifnot(n == sapply( n*(n-1)/2, function(n) sizeDiss(logical(n))))
```

---

summary.agnes   *Summary Method for 'agnes' Objects*

Description

Returns (and prints) a summary list for an agnes object. Printing gives more output than the corresponding [print.agnes](#) method.

Usage

```
## S3 method for class 'agnes'
summary(object, ...)
## S3 method for class 'summary.agnes'
print(x, ...)
```

Arguments

x, object   a [agnes](#) object.

...   potential further arguments (require by generic).

See Also

agnes, agnes.object.

Examples

```
data(agriculture)
summary(agnes(agriculture))
```

---

summary.clara   *Summary Method for 'clara' Objects*

Description

Returns (and prints) a summary list for a clara object. Printing gives more output than the corresponding print.clara method.

Usage

```
## S3 method for class 'clara'
summary(object, ...)
## S3 method for class 'summary.clara'
print(x, ...)
```

Arguments

x, object   a clara object.

...   potential further arguments (require by generic).

See Also

clara.object

Examples

```
## generate 2000 objects, divided into 5 clusters.
set.seed(47)
x <- rbind(cbind(rnorm(400, 0,4), rnorm(400, 0,4)),
           cbind(rnorm(400,10,8), rnorm(400,40,6)),
           cbind(rnorm(400,30,4), rnorm(400, 0,4)),
           cbind(rnorm(400,40,4), rnorm(400,20,2)),
           cbind(rnorm(400,50,4), rnorm(400,50,4))
)
clx5 <- clara(x, 5)
## Mis'classification' table:
table(rep(1:5, rep(400,5)), clx5$clustering) # -> 1 "error"
summary(clx5)

## Graphically:
```

```
par(mfrow = c(3,1), mgp = c(1.5, 0.6, 0), mar = par("mar") - c(0,0,2,0))
plot(x, col = rep(2:6, rep(400,5)))
plot(clx5)
```

---

summary.diana   *Summary Method for 'diana' Objects*

Description

Returns (and prints) a summary list for a diana object.

Usage

```
## S3 method for class 'diana'
summary(object, ...)
## S3 method for class 'summary.diana'
print(x, ...)
```

Arguments

x, object   a [diana](#) object.

...   potential further arguments (require by generic).

See Also

[diana](#), [diana.object](#).

---

summary.mona   *Summary Method for 'mona' Objects*

Description

Returns (and prints) a summary list for a mona object.

Usage

```
## S3 method for class 'mona'
summary(object, ...)
## S3 method for class 'summary.mona'
print(x, ...)
```

Arguments

x, object   a [mona](#) object.

...   potential further arguments (require by generic).

See Also

mona, mona.object.

---

summary.pam   *Summary Method for PAM Objects*

Description

Summarize a pam object and return an object of class summary.pam. There's a print method for the latter.

Usage

```
## S3 method for class 'pam'
summary(object, ...)
## S3 method for class 'summary.pam'
print(x, ...)
```

Arguments

x, object   a pam object.

...   potential further arguments (require by generic).

See Also

pam, pam.object.

---

twins.object   *Hierarchical Clustering Object*

Description

The objects of class "twins" represent an agglomerative or divisive (polythetic) hierarchical clustering of a dataset.

Value

See agnes.object and diana.object for details.

GENERATION

This class of objects is returned from agnes or diana.

METHODS

The "twins" class has a method for the following generic function: pltree.

INHERITANCE

The following classes inherit from class "twins" : "agnes" and "diana".

See Also

agnes,diana.

---

volume.ellipsoid   *Compute the Volume (of an Ellipsoid)*

Description

Compute the volume of geometric R object. This is a generic function and has a method for ellipsoid objects (typically resulting from ellipsoidhull().

Usage

```
volume(object, ...)
## S3 method for class 'ellipsoid'
volume(object, log = FALSE, ...)
```

Arguments

object	an R object the volume of which is wanted; for the ellipsoid method, an object of that class (see ellipsoidhull or the example below).
log	logical indicating if the volume should be returned in log scale. Maybe needed in largish dimensions.
...	potential further arguments of methods, e.g. log.

Value

a number, the volume V (or $\log(V)$ if log = TRUE) of the given object.

Author(s)

Martin Maechler (2002, extracting from former clusplot code); Keefe Murphy (2019) provided code for dimensions $d > 2$.

See Also

ellipsoidhull for spanning ellipsoid computation.

Examples

```
## example(ellipsoidhull) # which defines 'ellipsoid' object <namefoo>

myEl <- structure(list(cov = rbind(c(3,1),1:2), loc = c(0,0), d2 = 10),
                   class = "ellipsoid")
volume(myEl)# i.e. "area" here (d = 2)
myEl # also mentions the "volume"

set.seed(1)
d5 <- matrix(rt(500, df=3), 100,5)
e5 <- ellipsoidhull(d5)
```

---

votes.repub   *Votes for Republican Candidate in Presidential Elections*

Description

A data frame with the percents of votes given to the republican candidate in presidential elections from 1856 to 1976. Rows represent the 50 states, and columns the 31 elections.

Usage

```
data(votes.repub)
```

Source

S. Peterson (1973): *A Statistical History of the American Presidential Elections*. New York: Frederick Ungar Publishing Co.

Data from 1964 to 1976 is from R. M. Scammon, *American Votes 12*, Congressional Quarterly.

---

xclara   *Bivariate Data Set with 3 Clusters*

Description

An artificial data set consisting of 3000 points in 3 quite well-separated clusters.

Usage

```
data(xclara)
```

Format

A data frame with 3000 observations on 2 numeric variables (named V1 and V2) giving the x and y coordinates of the points, respectively.

Note

Our version of the xclara is slightly more rounded than the one from read.table(″xclara.dat″) and the relative difference measured by all.equal is 1.15e-7 for V1 and 1.17e-7 for V2 which suggests that our version has been the result of a options(digits = 7) formatting.

Previously (before May 2017), it was claimed the three cluster were each of size 1000, which is clearly wrong. pam(*, 3) gives cluster sizes of 899, 1149, and 952, which apart from seven "outliers" (or "mislabellings") correspond to observation indices {1 : 900}, {901 : 2050}, and {2051 : 3000}, see the example.

Source

Sample data set accompanying the reference below (file 'xclara.dat' in side 'clus_examples.tar.gz').

References

Anja Struyf, Mia Hubert & Peter J. Rousseeuw (1996) Clustering in an Object-Oriented Environment. *Journal of Statistical Software* **1**. doi:10.18637/jss.v001.i04

Examples

```
## Visualization: Assuming groups are defined as {1:1000}, {1001:2000}, {2001:3000}
plot(xclara, cex = 3/4, col = rep(1:3, each=1000))
p.ID <- c(78, 1411, 2535) ## PAM's medoid indices  == pam(xclara, 3)$id.med
text(xclara[p.ID,], labels = 1:3, cex=2, col=1:3)

 px <- pam(xclara, 3) ## takes ~2 seconds
 cxcl <- px$clustering ; iCl <- split(seq_along(cxcl), cxcl)
 boxplot(iCl, range = 0.7, horizontal=TRUE,
         main = "Indices of the 3 clusters of  pam(xclara, 3)")

 ## Look more closely now:
 bxCl <- boxplot(iCl, range = 0.7, plot=FALSE)
 ## We see 3 + 2 + 2 = 7  clear "outlier"s  or "wrong group" observations:
 with(bxCl, rbind(out, group))
 ## out    1038 1451 1610   30  327  562  770
 ## group    1    1    1    2    2    3    3
 ## Apart from these, what are the robust ranges of indices? -- Robust range:
 t(iR <- bxCl$stats[c(1,5),])
 ##     1  900
 ##   901 2050
 ## 2051 3000
 gc <- adjustcolor("gray20",1/2)
 abline(v = iR, col = gc, lty=3)
 axis(3, at = c(0, iR[2,]), padj = 1.2, col=gc, col.axis=gc)
```

Index