

# Examples from Multilevel Software Comparative Reviews

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## Abstract

The Center for Multilevel Modelling at the Institute of Education, London maintains a web site of “Software reviews of multilevel modeling packages”. The data sets discussed in the reviews are available at this web site. We have incorporated these data sets in the `lme4` package for R and, in this vignette, provide the results of fitting several models to these data sets.

```
Attaching package: 'lme4'
```

```
The following object(s) are masked from package:stats :
```

```
contr.SAS
```

## 1 Introduction

## 2 Two-level normal models

The Exam data set is used in fitting examples of two-level normal multilevel models.

```
> str(Exam)
`data.frame`:      4059 obs. of  10 variables:
 $ school  : Factor w/ 65 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 ...
 $ normexam: num   0.261  0.134 -1.724  0.968  0.544 ...
 $ schgend : Factor w/ 3 levels "mixed","boys",...: 1 1 1 1 1 1 1 1 1 ...
```

```

$ schavg : num 0.166 0.166 0.166 0.166 0.166 ...
$ vr : Factor w/ 3 levels "bottom 25%","mid 50%",...: 2 2 2 2 2 2 2 2 2 ...
$ intake : Factor w/ 3 levels "bottom 25%","mid 50%",...: 1 2 3 2 2 1 3 2 2 3 ...
$ standLRT: num 0.619 0.206 -1.365 0.206 0.371 ...
$ sex : Factor w/ 2 levels "F","M": 1 1 2 1 1 2 2 2 1 2 ...
$ type : Factor w/ 2 levels "Mxd","Sngl": 1 1 1 1 1 1 1 1 1 ...
$ student : Factor w/ 650 levels "1","2","3","4",...: 143 145 142 141 138 155 158 115 117 113 ...
> system.time(Eml <- lmer(normexam ~ standLRT + sex + schgend +
+ (1 | school), Exam), gc = TRUE)
[1] 0.12 0.00 0.23 0.00 0.00
> summary(Eml)
Linear mixed-effects model fit by REML
Formula: normexam ~ standLRT + sex + schgend + (1 | school)
Data: Exam
      AIC      BIC    logLik MLdeviance REMLdeviance
9361.673 9405.834 -4673.837  9325.501    9347.673
Random effects:
Groups Name Variance Std.Dev.
school (Intercept) 0.085829 0.29297
Residual 0.562534 0.75002
# of obs: 4059, groups: school, 65

Fixed effects:
              Estimate Std. Error DF t value Pr(>|t|)
(Intercept) -1.0493e-03 5.5569e-02 4054 -0.0189 0.98494
standLRT      5.5975e-01 1.2450e-02 4054 44.9601 < 2.2e-16
sexM          -1.6739e-01 3.4100e-02 4054 -4.9089 9.519e-07
schgendboys   1.7769e-01 1.1347e-01 4054 1.5659 0.11745
schgendgirls  1.5900e-01 8.9403e-02 4054 1.7784 0.07541

Correlation of Fixed Effects:
      (Intr) stnLRT sexM schgndb
standLRT -0.014
sexM      -0.316 0.061
schgendboys -0.395 -0.003 -0.145
schgendgrls -0.622 0.009 0.197 0.245

```

There are some interesting aspects of data management that show up in the analysis of these data. The student variable is an identifier of the student within the school. It would be best to combine the indicators of school and student to get a unique identifier of the student.

```

> Exam$ids <- with(Exam, school:student)[, drop = TRUE]
> str(Exam)
`data.frame`:      4059 obs. of  11 variables:
 $ school : Factor w/ 65 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 ...
 $ normexam: num 0.261 0.134 -1.724 0.968 0.544 ...
 $ schgend : Factor w/ 3 levels "mixed","boys",...: 1 1 1 1 1 1 1 1 1 ...
 $ schavg : num 0.166 0.166 0.166 0.166 0.166 ...
 $ vr : Factor w/ 3 levels "bottom 25%","mid 50%",...: 2 2 2 2 2 2 2 2 2 ...
 $ intake : Factor w/ 3 levels "bottom 25%","mid 50%",...: 1 2 3 2 2 1 3 2 2 3 ...
 $ standLRT: num 0.619 0.206 -1.365 0.206 0.371 ...
 $ sex : Factor w/ 2 levels "F","M": 1 1 2 1 1 2 2 2 1 2 ...
 $ type : Factor w/ 2 levels "Mxd","Sngl": 1 1 1 1 1 1 1 1 1 ...
 $ student : Factor w/ 650 levels "1","2","3","4",...: 143 145 142 141 138 155 158 115 117 113 ...
 $ ids : Factor w/ 4055 levels "1:1","1:4","1:6",...: 48 49 47 46 45 50 51 39 40 38 ...

```

Notice that there are 4059 observations but only 4055 unique levels of student within school. We can check the ones that are duplicated

```
> Exam$ids[which(duplicated(Exam$ids))]  
[1] 43:86 50:39 52:2 52:21  
4055 Levels: 1:1 1:4 1:6 1:7 1:13 1:14 1:16 1:17 1:19 1:22 1:27 ... 65:155
```

One of these duplicated cases is particularly interesting. One of the students with the duplicated student id 86 in school 43 is the only male student in this mixed school. This is probably a case of a misrecorded school.

### 3 Three-level Normal Models

These results are from the 1997 A-level Chemistry exam. The `school` is nested in `lea` (local education authority) and has unique levels for each of the 2410 schools. It is a good practice to make the nesting explicit by specifying the grouping factors as the ‘outer’ factor, `lea` in this case, and the interaction of the outer and inner factors, `lea:school` or `school:lea` in this case. This will ensure unique levels for each `school` within `lea` combination.

To fit the model `mC2` we increase the number of EM iterations from its default of 20 to 40. Without this change the current version of the `optim` function in R will declare convergence to an incorrect optimum. By increasing the number of EM iterations we are able to get closer to the optimum before calling `optim` and converge to the correct value. The `optim` function will be patched so this change will not be needed in future versions of R.

Data from the 1997 A-level Chemistry exam are available as `Chem97`.

```
> str(Chem97)  
`data.frame':      31022 obs. of  8 variables:  
 $ lea      : Factor w/ 131 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...  
 $ school   : Factor w/ 2410 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...  
 $ student  : Factor w/ 31022 levels "1","2","3","4",...: 1 2 3 4 5 6 7 8 9 10 ...  
 $ score    : num  4 10 10 10 8 10 6 8 4 10 ...  
 $ gender   : Factor w/ 2 levels "M","F": 2 2 2 2 2 2 2 2 2 2 ...  
 $ age      : num  3 -3 -4 -2 -1 4 1 4 3 0 ...  
 $ gcsescore: num  6.62 7.62 7.25 7.50 6.44 ...  
 $ gcsecnt  : num  0.339 1.339 0.964 1.214 0.158 ...  
  
> system.time(mC1 <- lmer(score ~ 1 + (1 | lea:school) + (1 |  
+ lea), Chem97), gc = TRUE)  
[1] 3.83 0.08 7.93 0.00 0.00  
  
> summary(mC1)
```

```

Linear mixed-effects model fit by REML
Formula: score ~ 1 + (1 | lea:school) + (1 | lea)
Data: Chem97
      AIC      BIC    logLik MLdeviance REMLdeviance
157881.8 157915.2 -78936.9   157869.9    157873.8
Random effects:
Groups      Name      Variance Std.Dev.
lea:school (Intercept) 2.74981  1.6583
lea        (Intercept) 0.15343  0.3917
Residual                        8.51591  2.9182
# of obs: 31022, groups: lea:school, 2410; lea, 131

Fixed effects:
              Estimate Std. Error    DF t value Pr(>|t|)
(Intercept) 5.3189e+00 5.8108e-02 31021  91.536 < 2.2e-16
> system.time(mC2 <- lmer(score ~ gcsecnt + (1 | school) +
+   (1 | lea), Chem97, control = list(niterEM = 40)), gc = TRUE)
[1] 1.17 0.00 2.31 0.00 0.00
> summary(mC2)
Linear mixed-effects model fit by REML
Formula: score ~ gcsecnt + (1 | school) + (1 | lea)
Data: Chem97
      AIC      BIC    logLik MLdeviance REMLdeviance
141707.2 141748.9 -70848.58   141685.8    141697.2
Random effects:
Groups      Name      Variance Std.Dev.
school      (Intercept) 1.163183 1.07851
lea         (Intercept) 0.020849 0.14439
Residual                        5.153861 2.27021
# of obs: 31022, groups: school, 2410; lea, 131

Fixed effects:
              Estimate Std. Error    DF t value Pr(>|t|)
(Intercept) 5.6377e+00 3.2353e-02 31020  174.26 < 2.2e-16
gcsecnt      2.4726e+00 1.6907e-02 31020  146.25 < 2.2e-16

Correlation of Fixed Effects:
      (Intr)
gcsecnt 0.056

```

## 4 Two-level models for binary data

The data frame `Contraception` provides data from the Bangladesh fertility survey.

```

> str(Contraception)
`data.frame`:      1934 obs. of  6 variables:
 $ woman   : Factor w/ 1934 levels "1","2","3","4",...: 1 2 3 4 5 6 7 8 9 10 ...
 $ district: Factor w/ 60 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ use      : Factor w/ 2 levels "N","Y": 1 1 1 1 1 1 1 1 1 1 ...
 $ livch    : Factor w/ 4 levels "0","1","2","3+": 4 1 3 4 1 1 4 4 2 4 ...
 $ age      : num  18.44 -5.56 1.44 8.44 -13.56 ...
 $ urban     : Factor w/ 2 levels "N","Y": 2 2 2 2 2 2 2 2 2 2 ...

```

```
> summary(Contraception[, -1])
```

	district	use	livch	age	urban
14	: 118	N:1175	0 :530	Min. :-13.560000	N:1372
1	: 117	Y: 759	1 :356	1st Qu.: -7.559900	Y: 562
46	: 86		2 :305	Median : -1.559900	
25	: 67		3+:743	Mean : 0.002198	
6	: 65			3rd Qu.: 6.440000	
30	: 61			Max. : 19.440000	
(Other):1420					

## 5 Growth curve model for repeated measures data

```
> str(Oxboys)
```

```
`data.frame':      234 obs. of  4 variables:
 $ Subject : Factor w/ 26 levels "1","10","11",...: 1 1 1 1 1 1 1 1 1 12 ...
 $ age     : num  -1.0000 -0.7479 -0.4630 -0.1643 -0.0027 ...
 $ height  : num  140 143 145 147 148 ...
 $ Occasion: Factor w/ 9 levels "1","2","3","4",...: 1 2 3 4 5 6 7 8 9 1 ...
- attr(*, "ginfo")=List of 7
 ..$ formula      :Class 'formula' length 3 height ~ age | Subject
 .. ..- attr(*, ".Environment")=length 4 <environment>
 ..$ order.groups: logi TRUE
 ..$ FUN          :function (x)
 .. ..- attr(*, "source")= chr "function (x) max(x, na.rm = TRUE)"
 ..$ outer        : NULL
 ..$ inner         : NULL
 ..$ labels       :List of 2
 .. ..$ age       : chr "Centered age"
 .. ..$ height    : chr "Height"
 ..$ units        :List of 1
 .. ..$ height    : chr "(cm)"

> system.time(mX1 <- lmer(height ~ age + I(age^2) + I(age^3) +
+ I(age^4) + (age + I(age^2) | Subject), Oxboys), gc = TRUE)

[1] 0.40 0.01 0.82 0.00 0.00

> summary(mX1)
```

```
Linear mixed-effects model fit by REML
Formula: height ~ age + I(age^2) + I(age^3) + I(age^4) + (age + I(age^2) | Subject)
Data: Oxboys
      AIC      BIC    logLik MLdeviance REMLdeviance
651.9081 693.372 -313.9541  625.3593    627.9081

Random effects:
Groups   Name              Variance Std.Dev. Corr
Subject (Intercept) 64.03130 8.00196
          age        2.86408 1.69236  0.614
          I(age^2)   0.67428 0.82115  0.215 0.658
Residual              0.21738 0.46624

# of obs: 234, groups: Subject, 26

Fixed effects:
              Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept) 149.01887    1.57032 229 94.8971 < 2.2e-16
```

```

age          6.17418      0.35650 229 17.3190 < 2.2e-16
I(age^2)     1.12823      0.35144 229  3.2103  0.001516
I(age^3)     0.45385      0.16246 229  2.7937  0.005653
I(age^4)    -0.37690      0.30018 229 -1.2556  0.210554

Correlation of Fixed Effects:
      (Intr) age      I(g^2) I(g^3)
age          0.572
I(age^2)     0.076  0.264
I(age^3)    -0.001 -0.340  0.025
I(age^4)     0.021  0.016 -0.857 -0.021

> system.time(mX2 <- lmer(height ~ poly(age, 4) + (age + I(age^2) |
+   Subject), Oxboys), gc = TRUE)

[1] 0.39 0.00 0.82 0.00 0.00

> summary(mX2)

Linear mixed-effects model fit by REML
Formula: height ~ poly(age, 4) + (age + I(age^2) | Subject)
Data: Oxboys
      AIC      BIC    logLik MLdeviance REMLdeviance
640.8686 682.3324 -308.4343  625.3593    616.8686

Random effects:
Groups   Name      Variance Std.Dev. Corr
Subject  (Intercept) 64.03114 8.00195
          age         2.86407 1.69236  0.614
          I(age^2)    0.67428 0.82115  0.215 0.658
Residual                    0.21738 0.46624

# of obs: 234, groups: Subject, 26

Fixed effects:
      Estimate Std. Error DF t value Pr(>|t|)
(Intercept)  149.51976    1.59026 229 94.0222 < 2.2e-16
poly(age, 4)1  64.54095    3.32780 229 19.3945 < 2.2e-16
poly(age, 4)2   4.20322    1.02361 229  4.1063 5.597e-05
poly(age, 4)3   1.29077    0.46628 229  2.7682 0.006098
poly(age, 4)4  -0.58547    0.46630 229 -1.2556 0.210554

Correlation of Fixed Effects:
      (Intr) p(,4)1 p(,4)2 p(,4)3
poly(ag,4)1 0.631
poly(ag,4)2 0.230  0.583
poly(ag,4)3 0.000  0.000  0.000
poly(ag,4)4 0.000  0.000  0.000  0.000

```

## 6 Cross-classification model

```

> str(ScotsSec)

`data.frame':      3435 obs. of  6 variables:
 $ verbal : num  11 0 -14 -6 -30 -17 -17 -11 -9 -19 ...
 $ attain : num  10 3 2 3 2 2 4 6 4 2 ...
 $ primary: Factor w/ 148 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ sex    : Factor w/ 2 levels "M","F": 1 2 1 1 2 2 2 1 1 1 ...
 $ social : num   0 0 0 20 0 0 0 0 0 0 ...
 $ second : Factor w/ 19 levels "1","2","3","4",...: 9 9 9 9 9 9 1 1 9 9 ...

```

```

> system.time(mS1 <- lmer(attain ~ sex + (1 | primary) + (1 |
+   second), ScotsSec), gc = TRUE)
[1] 0.21 0.01 0.42 0.00 0.00
> summary(mS1)

Linear mixed-effects model fit by REML
Formula: attain ~ sex + (1 | primary) + (1 | second)
Data: ScotsSec
      AIC      BIC    logLik MLdeviance REMLdeviance
17137.91 17168.62 -8563.956   17123.49     17127.91
Random effects:
Groups   Name      Variance Std.Dev.
primary (Intercept) 1.10962   1.0534
second  (Intercept) 0.36966   0.6080
Residual                    8.05511  2.8382
# of obs: 3435, groups: primary, 148; second, 19

Fixed effects:
              Estimate Std. Error   DF t value Pr(>|t|)
(Intercept) 5.2552e+00 1.8432e-01 3433  28.5107 < 2.2e-16
sexF         4.9851e-01 9.8255e-02 3433   5.0737 4.109e-07

Correlation of Fixed Effects:
      (Intr)
sexF -0.264

```